ROTATION-DRIVEN FLEXIBLE SPARSE ARRAYS FOR HIGH-RESOLUTION DOA ESTIMATION

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ABSTRACT

This paper presents a novel flexible sparse antenna array (FSAA) design that improves angular resolution in direction-of-arrival (DOA) estimation. In contrast to conventional fixed arrays, the proposed FSAA exploits rotational flexibility and offers additional degrees of freedom. We present a joint DOA estimation and array rotation optimization problem with the goal of minimizing the output power of a minimum-variance distortionless response (MVDR) beamformer. Although a closed-form solution of the optimal rotation angle for given source DOAs and important insight on its impact on the beamforming performance can be obtained for the case of two sources, under certain conditions, the general problem remains challenging. We find the optimum rotation angle by maximizing the values of the peaks of the MVDR power spectrum. Furthermore, we also present a particle swarm optimization-based technique that solves the proposed problem efficiently by maximizing the harmonic mean of the MVDR spectrum peaks. The optimized array rotation improves DOA estimation accuracy with the same number of sensors and without changing the inter-sensor spacing. Both theoretical analysis and simulation results demonstrate that FSAA promisingly decreases DOA estimation errors, especially under low signal-to-noise ratio (SNR) scenarios.

Index Terms— DOA estimation, flexible antenna array, sparse arrays.

1. INTRODUCTION

High-resolution direction-of-arrival (DOA) estimation is critical in array signal processing [1], [2]. Classical techniques such as multiple signal classification (MUSIC) [3], minimum variance distortionless response (MVDR) [4], [5], estimation of signal parameters via rotational invariance techniques (ESPRIT) [6], and compressive sensing [7] achieve high angular resolution provided that the input signal-to-noise ratio (SNR) is high and the array offers a sufficiently large aperture.

Sparse arrays are a type of antenna structures that extend the effective aperture due to virtual antenna sensors and offer additional degrees of freedom (DoF) with fewer sensor elements that are unevenly spaced [8], [9]. Minimum redundancy arrays (MRA) [10], coprime arrays [11], and nested arrays [12] are the well-known types of sparse arrays that are commonly used for DOA estimation. In an ideal scenario, a sparse array with D sensors can detect up to $O(D^2)$ uncorrelated sources, while a uniform array with the same number of sensors can only resolve up to $D-1$ sources [13]. However, when their geometries are fixed, the resolving power is limited, particularly in challenging scenarios such as closely spaced sources and low SNR conditions.

Flexible antenna arrays (FAAs) have emerged as an encouraging solution capable of adjusting to irregular shapes while sustaining superior electromagnetic performance [14]. The ability to fold, bend, rotate, and twist makes FAA exceptionally advantageous in dynamic configurations. Recent research highlights that FAAs enhance DOA estimation by enabling mechanical flexibility, thus strengthening adaptive beamforming [15]. The flexible extended nested array (FENA) aims to offer additional DoFs and overcome holes in the difference coarrays. However, FENA provides design-time adaptability only; it lacks real-time reconfiguration (e.g., rotation, bending, folding) that could further improve DOA performance during operation. Furthermore, recent studies also highlight fluid antennas that help improve DOA estimation performance through optimizing sensor positions [16]. One major issue with fluid antennas is the need for complicated controllers that can move sensor elements adaptively, which results in changing the physical aperture of the array.

Flexible rotation allows sensor arrays to make dynamic adjustments in varying scenarios, avoiding complicated electronic beamforming and feed networks [17]. Rotation changes the array's effective aperture and improves its spatial response and beam pattern in the desired directions, thus providing superior angular resolution and DOA estimation performance [18]. Recent studies [19] proposed element-wise sensor rotations in a dipole linear array to estimate DOAs and polarization, reducing horizontal-polarization ambiguity. Although the above-mentioned studies discuss array rotation, there is still significant potential for improvements in identifying a closed-form solution to this problem. Minimizing computational complexity and the use of a global array-based rotation instead of element-wise rotation could promisingly

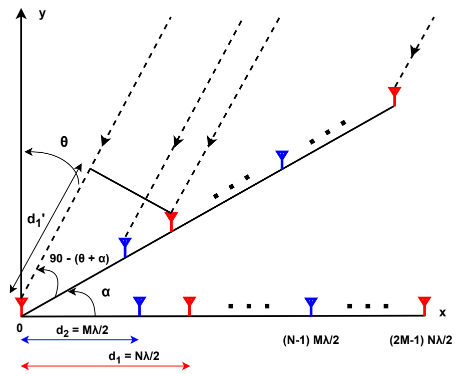

Fig. 1. Graphical representation of the FSAA rotation

improve its feasibility in real-time applications.

In this paper, we address an optimization problem for maximizing MVDR peak magnitudes via flexible array rotation. We aim to improve DOA estimation performance without changing the physical aperture or relative sensor positions while ensuring adaptability to real-time scenarios. We introduce a flexible sparse antenna array (FSAA) framework that provides additional DoFs [1] through rotational flexibility that tries to achieve orthogonality between closely spaced steering vectors. We present a rotation-parameterized steering vector that reveals the dependence of angular resolution capability and DOA estimation performance on array orientation. Next, we jointly estimate DOAs and the optimal rotation angle ($\alpha_{\text{opt}}(\boldsymbol{\theta})$) by searching the peaks in the MVDR power spectrum. To overcome the high computational complexity associated with the grid-based search for the $\alpha_{\text{opt}}(\boldsymbol{\theta})$, we propose a particle swarm optimization (PSO) technique that maximizes the harmonic mean (HM) of MVDR spectrum peaks.

Notations: Bold uppercase/lowercase letters represent matrices/vectors. The superscripts $(\cdot)^T$ and $(\cdot)^H$ denote transpose and complex conjugate transpose, respectively. $\mathbb{E}\{\cdot\}$ and $\mathbf{I}_D$ denote mathematical expectations and the $D \times D$ identity matrix, respectively. The notation $\mathbb{C}^{M\times M}$ represents the space of $M \times M$ complex-valued matrices.

2. SYSTEM MODEL

2.1. Sparse Arrays

We consider a coprime linear array (CLA), a sparse array constructed by two uniform linear arrays (ULAs) with distinct inter-element spacings, as shown in Fig. 1 [20]. The first ULA has $2M$ sensors spaced by $d_1 = Nd$, while the second ULA has N sensors spaced by $d_2 = Md$. Here, M and N are coprime integers, and $d = \lambda/2$ represents half-wavelength spacing. The physical sensor positions of the CLA are defined as

$$\mathcal{Q}_C = \{mNd \mid 0 \le m \le 2M-1\} \cup \{nMd \mid 0 \le n \le N-1\}. \tag{1}$$

The first sensor of each ULA serves as a common reference, resulting in a total of $D = 2M + N - 1$ physical sensors.

Similarly, a standard sparse nested array (NA) is constructed from two ULAs with different inter-element spacings [12]. The first ULA (dense) has M sensor elements with spacing $d = \lambda/2$, while the second ULA has N sensor elements with spacing $(M + 1)d$. The union of these two ULAs defines the physical sensor locations of the nested array, which are given by

$$\mathcal{Q}_N = \{0, 1, \dots, (M-1)\}d \cup \{(M+1), \dots, N(M+1)\}d. \tag{2}$$

The resulting NA has $D = M + N$ physical sensors with no co-located sensor positions between the two subarrays.

2.2. Signal Model

Consider L uncorrelated narrowband far-field signals $\{s_l(t)\}_{l=1}^{L}$ that impinge on the sparse antenna array (SAA) [2] from azimuth angles $\boldsymbol{\theta} = [\theta_1, \theta_2, \dots, \theta_L]^T$. Let $\mathbf{q} = [q_1, q_2, \dots, q_D]^T$ denote the actual sensor positions with $q_1 = 0$. The SAA steering vector for θ_l can be expressed as

$$\mathbf{a}(\theta_l) = \left[1, e^{j\frac{2\pi}{\lambda}q_2 \sin(\theta_l)}, \dots, e^{j\frac{2\pi}{\lambda}q_D \sin(\theta_l)}\right]^T. \tag{3}$$

The received signal at time t is given by

$$\mathbf{x}(t) = \mathbf{A}(\boldsymbol{\theta})\mathbf{s}(t) + \mathbf{n}(t), \tag{4}$$

where $\mathbf{A}(\boldsymbol{\theta}) = [\mathbf{a}(\theta_1), \dots, \mathbf{a}(\theta_L)] \in \mathbb{C}^{D\times L}$, $\mathbf{s}(t) \in \mathbb{C}^{L\times 1}$ is the sources signal vector, and $\mathbf{n}(t) \sim \mathcal{CN}(0, \sigma_n^2\mathbf{I}_D)$ is additive white Gaussian noise with zero-mean and covariance $\sigma_n^2\mathbf{I}_D$. The covariance matrix of the received signal vector $\mathbf{x}(t)$ can be written as

$$\mathbf{R}_{xx} = \mathbb{E}[\mathbf{x}(t)\mathbf{x}^H(t)] = \mathbf{A}(\boldsymbol{\theta})\mathbf{R}_{ss}\mathbf{A}^H(\boldsymbol{\theta}) + \sigma_n^2\mathbf{I}_D. \tag{5}$$

Here, $\mathbf{R}_{ss} = \text{diag}(\rho_1, \dots, \rho_L)$ and $\rho_l = \mathbb{E}[|s_l(t)|^2]$ is the power of the l-th signal. Practically, $\mathbf{R}_{xx}$ can be estimated as

$$\hat{\mathbf{R}}_{xx} = \frac{1}{T}\sum_{t=1}^{T}\mathbf{x}(t)\mathbf{x}^H(t), \tag{6}$$

where T is the number of snapshots. Adaptive beamformers such as MVDR rely on $\hat{\mathbf{R}}_{xx}$ for DOA estimation. In SAA, achieving high-resolution requires accurate covariance estimation, which becomes challenging at low SNR or with limited snapshots. To address this issue, the proposed FSAA leverages flexible array rotation and tries to achieve orthogonality between closely spaced steering vectors by providing additional DoFs, thereby improving the DOA estimation with reduced estimation errors.

[1] The additional DoFs are discussed in terms of array rotational flexibility rather than increased virtual antenna positions or reduced coarray holes. The rotational flexibility helps reduce correlations among the steering vectors and promotes their orthogonality.

[2] In this paper, SAA is used generically to refer both CLA and NA.

3. PROPOSED FSAA ROTATION

We consider the SAA in Fig. 1 with the received signal vector defined in (4). The fixed SAA faces difficulty in resolving closely spaced off-broadside sources. To improve the angular resolution and detection capability of the SAA, we propose an FSAA that rotates over a range of 0° to 360°, making the DOA estimation an explicit function of the rotation angle. When the array is rotated counterclockwise by a rotation angle α, the phase response of each antenna sensor relative to the reference sensor at $(0,0)$ changes. The resulting path-length difference (d'_1) in the rotated FSAA for the antenna sensor, initially located at $d_1 = Nd$, is given by

$$\cos(90^\circ \ - (\theta+\alpha)) = \frac{d'_1}{d_1} \Rightarrow d'_1 = d_1 \sin(\theta+\alpha). \quad (7)$$

Using (7), the FSAA steering vector for a source signal arriving from θ can be expressed as

$$\mathbf{a}(\theta,\alpha) = \left[1, e^{j\frac{2\pi}{\lambda}q_2\sin(\theta+\alpha)}, \ldots, e^{j\frac{2\pi}{\lambda}q_D\sin(\theta+\alpha)}\right]^T. \quad (8)$$

3.1. *MVDR Peaks Analysis for Optimal Rotation*

The MVDR beamformer minimizes the array output power while ensuring a distortionless response in the desired look direction. The fixed array geometry, with $\alpha = 0^\circ$, has limited DOFs, and consequently closely spaced sources cannot be reliably detected. Therefore, we optimize α minimizing the MVDR power spectrum, which is given by

$$P_{\mathrm{MVDR}}(\theta,\alpha) = \frac{1}{\mathbf{a}(\theta,\alpha)^H\,\hat{\mathbf{R}}_{xx}^{-1}(\alpha)\,\mathbf{a}(\theta,\alpha)}, \quad (9)$$

where $\hat{\mathbf{R}}_{xx}^{-1}(\alpha)$ indicates that the sample covariance matrix is a function of α. The optimal solution of α can therefore be obtained as:

$$\alpha_{\mathrm{opt}}(\boldsymbol{\theta}) = \arg\max_{\alpha}\ (\mathbf{a}^H(\theta,\alpha)\hat{\mathbf{R}}_{xx}^{-1}(\alpha)\mathbf{a}(\theta,\alpha)). \quad (10)$$

To our knowledge, the analytical solution for the optimum α is not tractable, because $\mathbf{a}(\theta,\alpha)$ is already a complicated function of α and $\hat{\mathbf{R}}_{xx}^{-1}$ is also a function of α. Under certain conditions, such as when there are two sources, the sample covariance matrix approaches the true covariance matrix, the array is uniform linear, and the MVDR spectrum can be approximated with the Bartlett power spectrum, a closed form expression of α can be derived. Skipping the derivations due to space constraint, we show that the optimum α is such that

$$(\sin\theta_1 - \sin\theta_2)\cos\alpha + (\cos\theta_1 - \cos\theta_2)\sin\alpha = \frac{p\lambda}{D}, \quad (11)$$

where p is a non-zero integer. The solution of this equation gives the optimal α, which is given by

$$\alpha_{\mathrm{opt}}(\boldsymbol{\theta}) = \sin^{-1}\left(\frac{p\lambda}{D}\right) - \sin^{-1}\left(\frac{a_{12}}{\sqrt{a_{12}^2 + b_{12}^2}}\right), \quad (12)$$

where $a_{12} = \sin\theta_1 - \sin\theta_2$ and $b_{12} = \cos\theta_1 - \cos\theta_2$. It is worthwhile to mention that the optimum α turns out to be the one that minimizes $\mathbf{a}^H(\theta_1,\alpha)\mathbf{a}(\theta_2,\alpha)$, or, in other words, the one that maintains orthogonality between $\mathbf{a}(\theta_1,\alpha)$ and $\mathbf{a}(\theta_2,\alpha)$. Although the analysis of two sources, under the aforementioned conditions, provides insight into the role of optimization of α, the resulting α is a function of the source angles, which are unknown and need to be estimated. This means that an iterative approach becomes necessary, which iteratively estimates the source angles for a given α and optimizes α for the estimated source angles until convergence is achieved. We propose a grid-based search for α, which is not subject to any of the aforementioned conditions. More specifically, for a given α, L MVDR power spectrum peaks are found, and the α value that maximizes these peaks is chosen as the $\alpha_{\mathrm{opt}}(\boldsymbol{\theta})$. The optimally rotated FSAA now resolves closely spaced sources with low root mean-square error (RMSE). It should be noted that the proposed algorithm requires the calculation of $\hat{\mathbf{R}}_{xx}$ for each α, which can be challenging in practice. However, by training deep neural networks, the mapping between α and $\hat{\mathbf{R}}_{xx}$ can be effectively learned offline without requiring to calculate $\hat{\mathbf{R}}_{xx}$ online. This possibility will be explored in our extended version of this paper [21].

3.2. *PSO-Based Optimization for Optimal Rotation*

To determine the $\alpha_{\mathrm{opt}}(\boldsymbol{\theta})$ for DOA estimation, a grid-based search evaluates MVDR magnitudes over a finely spaced set of rotation angles. The computational complexity of this approach increases significantly with the resolution of the grid, making it prohibitive for real-time adaptive algorithms. To overcome this limitation, we propose the use of PSO, a heuristic global optimization technique, which efficiently determines $\alpha_{\mathrm{opt}}(\boldsymbol{\theta})$ by maximizing the HM of the L largest peaks in the MVDR spectrum, as formulated by the following optimization problem:

$$\alpha_{\mathrm{opt}}(\boldsymbol{\theta}) = \arg\max_{\alpha}\left\{\frac{L}{\sum_{l=1}^{L}\frac{1}{P_l^{\mathrm{FSAA}}(\theta,\alpha)}}\right\} \quad (13)$$

$$\text{s.t.} \qquad lb^\circ \le \alpha \le ub^\circ, \quad (14)$$

where $P_l^{\mathrm{FSAA}}(\theta,\alpha)$ represents the l-th largest peak magnitude in the MVDR spectrum for the given rotation angle. PSO iteratively updates a population (swarm) of candidate solutions (particles), which adjust their positions based on their best personal (P^{best}) and best global (G^{best}) positions. Both the position (X_i^k) and the velocity (V_i^k) of each particle at iteration $k+1$ can be expressed as [22].

$$X_i^{k+1} = X_i^k + V_i^{k+1}, \quad (15)$$

$$V_i^{k+1} = wV_i^k + c_1 r_1\left(P_i^{\mathrm{best}} - X_i^k\right) + c_2 r_2\left(G_i^{\mathrm{best}} - X_i^k\right). \quad (16)$$

Here, w denotes the inertia weight, and c_1 and c_2 represent the acceleration constants. The convergence of the PSO algorithm depends on the values of w, c_1, and c_2, which control the movement of each particle and are briefly discussed in [23]. Furthermore, r_1 and r_2 are uniformly distributed random values within the range of $[0, 1]$. Similarly, P_i^{best} is the best position found by the particle i, while G_i^{best} is the best position found by the swarm. The PSO algorithm significantly overcomes the computational complexity required in grid-based search when determining $\alpha_{\text{opt}}(\boldsymbol{\theta})$.

4. SIMULATION RESULTS

In this section, we test the proposed FSAA system with MVDR beamforming and compare its performance against standard fixed arrays. Simulations are performed for a range of SNR, but with a focus on SNR=5 dB to evaluate the robustness of the proposed approach in real-world scenarios. Unless otherwise specified, we use $T = 500$ snapshots and $B = 500$ Monte Carlo trials. The CLA uses a pair of coprime integers $(M, N) = (3, 5)$, and the nested array uses $(M, N) = (4, 6)$. The operating frequency is $f = 4$ GHz.

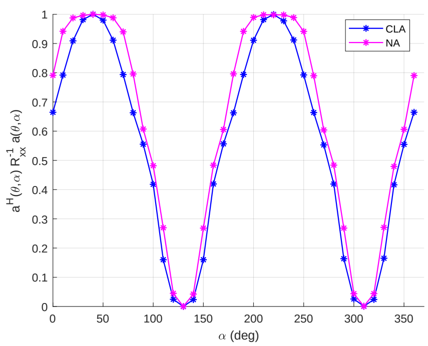

Fig. 2. MVDR magnitudes for L = 3 over 0° to 360° rotation

The maximum of the MVDR power spectrum is obtained for distinct array rotations, as illustrated in Fig. 2. The $\alpha_{\text{opt}}(\boldsymbol{\theta})$ value of the array corresponds to the angle with the maximum peak values (output of (10)). Due to the symmetry of the array, the maximum of the MVDR power spectrum occurs at two rotation angles ranging from 0° to 360°, as shown in Fig. 2. Furthermore, it can also be observed that this maximum value substantially decreases if the rotation angle is suboptimal. To overcome the high computational complexity of grid-based search, we employ PSO, which, upon convergence, achieves the same $\alpha_{\text{opt}}(\boldsymbol{\theta})$ with significantly reduced computation time. The corresponding convergence plot is skipped because of page limitation. The proposed PSO algorithm uses 24 particles and 20 iterations and converges in 1.2 seconds, while the grid-based search takes 8.9 seconds to determine $\alpha_{\text{opt}}(\boldsymbol{\theta})$. All simulations were run on MATLAB R2023b (Intel^R Core^{TM}, 3.60 GHz; 16 GB RAM; Windows 11).

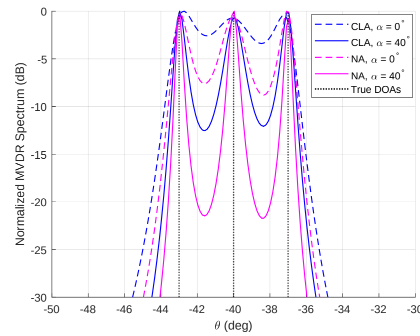

Fig. 3. MVDR spectrum with and without optimal rotation

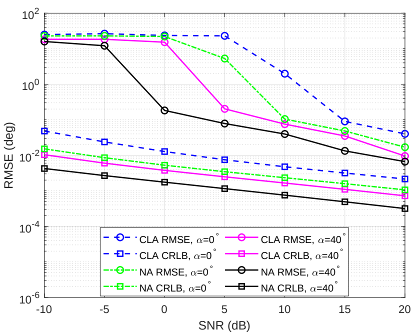

Fig. 4. RMSE versus CRLB at fixed and optimal rotation

We evaluate the performance of the proposed FSAA for L = 3 sources located at azimuth angles $[-43°, -40°, -37°]$, as shown in Fig. 3. For a fixed SAA, the array struggles to identify these sources, resulting in poor angular resolution of the source DOAs. To address this issue in a time-efficient manner, we employ PSO to determine $\alpha_{\text{opt}}(\boldsymbol{\theta})$, resulting in $\alpha_{\text{opt}}(\boldsymbol{\theta}) = 40°$ and $220°$, the same as the grid search. When the array is rotated by $\alpha_{\text{opt}}(\boldsymbol{\theta})$, all sources are detected with maximum efficiency, as shown in Fig. 3.

Similarly, the RMSE of the estimated DOAs is compared to Jansson Cramer-Rao lower bound (CRLB) [24] as a function of the SNR for L = 3 signals, as shown in Fig. 4. It is evident that the array with $\alpha_{\text{opt}}(\boldsymbol{\theta})$ results in minimal RMSE and CRLB, ensuring superior DOA estimation performance.

5. CONCLUSIONS

This paper introduced an FSAA framework that improves DOA estimation by introducing rotational flexibility to sparse antenna arrays without changing inter-sensor spacing or their count. The optimal rotation angle was obtained using grid search and PSO. The simulation results revealed that FSAA outperformed fixed sparse arrays, as it consistently lowered RMSE and approached theoretical bounds. The proposed FSAA offers additional DoFs by using rotational flexibility that promotes orthogonality among steering vectors, making it a strong candidate for next-generation sensing and radar applications that require high-resolution DOA estimation.

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