Optimizing the Ambiguity Function Profile of FHCS: A Sensing-Communication Trade-Off

Renjith Hari Kumar*, Elias Aboutanios*, Aboulnasr Hassanien†, Yimin D. Zhang‡ and Jinhong Yuan*
*School of Electrical Engineering and Telecommunications,UNSW, Sydney, New South Wales, Australia
†Department of Electrical Engineering, Wright State University, Dayton, OH 45435, USA
‡Department of Electrical and Computer Engineering, Temple University, Philadelphia, PA 19312, USA
E-mail: r.hari_kumar@unsw.edu.au, elias@ieee.org, hassanien@ieee.org, ydzhang@temple.edu, jinhong.yuan@unsw.edu.au

***Abstract*—Dual function radar communications (DFRC) alleviates the competition over the radio frequency spectrum by embedding communication symbols onto radar emissions. DFRC systems employing frequency hopping multiple-input multiple-output (MIMO) radar achieve higher bit rates through fast-time information embedding. One such scheme is the frequency hopping code selection (FHCS) which modulates frequency hops within each sub-pulse to transmit information. However, this modulation adversely impacts the sensing performance. In this work, we propose a strategy to control these losses by selectively excluding frequencies in each sub-pulse. This permits communication performance to be traded for improved ambiguity function, thus enhancing the sensing performance. Simulation results demonstrate the effectiveness of the proposed strategy.**

***Index Terms*—FH-MIMO radar, dual function radar communication, ambiguity function, frequency hopping code selection, sensing-communication trade-off**

I. Introduction

Dual function radar communication (DFRC) systems have emerged as an effective solution to the increasing competition over radio frequency (RF) spectrum, as they enable radar systems to simultaneously perform detection and communication tasks [1]–[6]. DFRC systems with radar operation as the primary objective seek to perform communications in a manner that incurs a tolerable cost to the radar while ensuring a minimum signal-to-interference-plus-noise ratio (SINR) at the communication receiver. Various DFRC schemes that embed communication symbols into a radar waveform have been proposed. Earlier DFRC systems introduced communication information into the sidelobes of the radar waveform while aiming to preserve the main lobe [7], [8]. In [9], phase modulation (PM) was used to achieve communications without altering the spectral profile of the radar signal. Multiple-input multiple-output (MIMO) radars [10], which provide increased number of degrees of freedom (DOFs), were then utilized to achieve higher data rates in DFRC systems [11]–[15].

The schemes above embed information in slow-time, which limits the data rate to a single communication symbol per radar pulse. As a result, the achievable bit rate is constrained by the pulse repetition frequency (PRF) of the radar. Embedding information in the fast time removes this limitation and unlocks higher data rates. To this end, frequency hopped MIMO (FH-MIMO) DFRC systems have been proposed in [16], [17] and further investigated in [18]–[20]. A generalized framework encompassing different information embedding strategies was also developed in [21]. Yet, the higher data rates enabled by fast-time information embedding come at the cost of increased sensing performance losses, as they require deviations of the radar transmit waveform from its optimal form [22]. From the radar perspective, the ambiguity function (AF), which is the output of the matched filter (MF) at the receiver, is a key measure for assessing the quality of the radar waveform [23] as it characterizes the range resolution. It is desirable that the AF exhibit a narrow main lobe and minimal sidelobe levels (SLLs) [24].

While the PSK-based FH-MIMO DFRC scheme [25], [26] achieves a reasonable AF, it is prone to out-of-band signal leakage due to phase discontinuities between adjacent hops. The frequency hopping code selection (FHCS) scheme [17], on the other hand, has excellent in-band confinement and eliminates the need for channel estimation, as information is encoded via the selection of the frequency hops. However, the FHCS scheme exhibits a poor average AF [17], [21], [27], a problem that needs to be addressed to ensure acceptable sensing performance. While several attempts have been made to address this problem [28], [29], the improvement is limited, particularly for systems with a higher number of antennas.

In this work, we develop an approach for trading off the performance of sensing and communications against each other. To this end, we propose a strategy that excludes a certain number of hops in each sub-pulse to reduce the number of overlapping hops between successive sub-pulses. This results in a distinct symbol dictionary being used in each sub-pulse, thus significantly reducing AF levels near the main lobe. As a result, the probability of target detection is improved.

The paper is organized as follows: in Section II we describe the signal model and discuss the AF. The limitations of the existing methods are summarized in Section III. In Section IV, we detail the new approach of excluding the hops to improve the AF. A separate symbol dictionary has to be designed for each sub-pulse in order to achieve this. Section V shows the simulation results. Section VI concludes this paper.

Notation:. The following notations are used here. Operations $\otimes$, $(\cdot)^*$, $(\cdot)^T$, and $(\cdot)^H$ indicate the Kronecker product, complex conjugate, transpose, and conjugate transpose, respectively. The symbol $\subseteq$ denotes the subset, $\cap$ denotes the set intersection and $\lfloor\cdot\rfloor$ represents the floor function.

979-8-3315-1213-2/25/$31.00 ©2025 IEEE

II. System Model

A. Frequency Hopping Code Selection

Consider an FH-MIMO DFRC system that uses M transmit and N receive co-located antennas and a single-antenna communication receiver. The radar pulse, of width T_w, is divided into Q sub-pulses or chips, each having a duration of Δ_t. The baseband representation of the FHCS transmitted signal in a direction θ is given by [21]

$$s(t;n) = \mathbf{a}^T(\theta)\boldsymbol{\phi}(t) = \mathbf{a}^T(\theta)\sum_{q=0}^{Q-1}\mathbf{S}_q^{(n)}\mathbf{h}(t)\Pi_q(t), \quad (1)$$

where $\mathbf{a}(\theta)$ is the transmit steering vector and $\mathbf{S}_q^{(n)}$ is the $M \times K$ selection matrix that specifies the choice, in the qth sub-pulse (also called chip) of the nth pulse, of M out of K available frequency hops. The $K \times 1$ vector of FH waveforms is given by,

$$\mathbf{h}(t) = \exp(j2\pi\mathbf{d}\Delta_f t) = [1, e^{j2\pi\Delta_f t}, \ldots, e^{j2\pi(K-1)\Delta_f t}]^T, \quad (2)$$

with $\mathbf{d} = [0, 1, \ldots, K-1]^T$ is a vector of frequency indices and $\Pi_q(t)$ is the rectangular function given by

$$\Pi_q(t) = \Pi(t - q\Delta_t) \triangleq \begin{cases} 1, & q\Delta_t \le t \le (q+1)\Delta_t, \\ 0, & \text{otherwise.} \end{cases} \quad (3)$$

Here $\Delta_f = BW/K$ is the hopping frequency increment and BW is the system bandwidth. The choice of M out of K hops results in $\binom{K}{M}$ combinations that are arranged into a codebook or symbol dictionary. Since the communication receiver has access to the complete dictionary, it is able to use a matched filter to decode the transmitted symbols.

B. The FH-MIMO Radar

Consider a target located in the far-field at a direction θ_r and having a reflection coefficient $\alpha(n)$. The FH-MIMO waveform from the mth transmit antenna in the nth pulse is given by

$$\phi_m(t;n) = \sum_{q=0}^{Q-1} e^{j2\pi c_q(m)\Delta_f t}\Pi_q(t), \quad (4)$$

where $c_q(m)$ is the index of the hop chosen for transmission on the mth element in the qth chip. For radar-only operation, the hops are fixed for all the pulses and are designed to generate a waveform with desirable AF properties.

The $N \times 1$ received signal vector at the radar receiver is

$$\mathbf{r}(t;n) = \alpha(n)\mathbf{b}(\theta_r)\mathbf{a}^T(\theta_r)\boldsymbol{\phi}(t-\tau;n)e^{j2\pi\nu t} + \mathbf{w}(t;n), \quad (5)$$

where τ denotes the round-trip delay to the target, ν is the Doppler frequency, $\mathbf{b}(\theta_r)$ is the steering vector at the radar receiver, and $\mathbf{w}(t;n)$ is additive noise. The Doppler frequency values are much smaller than the frequency hops and hence can be ignored in what follows. Using these assumptions and ignoring the noise, the extended signal vector at the MF output can be written as

$$\mathbf{y}(n) = \alpha(n)\mathbf{v}(\theta_r), \quad (6)$$

where

$$\mathbf{v}(\theta_r) = \text{vec}\left\{\mathbf{b}(\theta_r)\int_0^{T_w}\mathbf{a}^T(\theta_r)\boldsymbol{\phi}(t-\tau;n)\boldsymbol{\phi}^H(t;n)dt\right\} \quad (7)$$

is the $MN \times 1$ extended array steering vector and $\text{vec}\{\cdot\}$ denotes the vectorization operation that stacks the columns of a matrix into a column vector. The integral term in (6) is equivalent to the radar AF at zero Doppler. The cross ambiguity function (CAF) between two waveforms $\phi_{m_1}(t)$ and $\phi_{m_2}(t)$ is [30],

$$\chi_{m_1,m_2}(\tau,\nu) \triangleq \int_0^T \phi_{m_1}(t)\phi_{m_2}^*(t+\tau)e^{j2\pi\nu t}dt. \quad (8)$$

When $m_2 = m_1 = m$, $\chi_{m,m}(\tau,\nu)$ represents the auto ambiguity function (AAF) of waveform $\phi_m(t)$. The overall AF of the FH-MIMO radar is given by,

$$\chi_{MIMO}(\tau,\nu) = \sum_{m_1,m_2=1}^{M}\chi_{m_1,m_2}(\tau,\nu),\ \tau = i\Delta_t, i = 1,\ldots,Q. \quad (9)$$

In what follows, we omit ν in the AF expressions due to the zero-Doppler assumption.

III. Limitations of Existing Approaches

Let $\mathbf{c}_q$ denote the symbol transmitted in the qth chip. The CAF at integer delays, $\tau = k\Delta_t$, can be expressed as [29],

$$\chi_{m_1,m_2}(k\Delta_t) = \sum_{q=1}^{Q-k}\delta\Big(c_q(m_1), c_{q+k}(m_2)\Big), \quad (10)$$

where $\delta(\cdot)$ denotes the Kronecker Delta function. Substituting (10) into (9) and rearranging the summation, the overall AF at integer delays can be written as

$$\chi_{MIMO}(k\Delta_t) = \sum_{q=1}^{Q-k}\sum_{m_1,m_2=1}^{M}\delta\Big(c_q(m_1), c_{q+k}(m_2)\Big). \quad (11)$$

Now, let us define $\varsigma(\mathbf{c}_i, \mathbf{c}_l) = \sum_{m_1,m_2=1}^{M}\delta\left(c_i(m_1), c_l(m_2)\right)$ as the number of hops shared by the symbols $\mathbf{c}_i$ and $\mathbf{c}_l$. Then, we can express the overall AF as

$$\chi_{MIMO}(k\Delta_t) = \sum_{q=1}^{Q-k}\varsigma(\mathbf{c}_q, \mathbf{c}_{q+k}). \quad (12)$$

This expression gives the AF in terms of the symbols allowing us to quantify the impact of the information embedding on the radar performance. When $\mathbf{c}_{q+k} = \mathbf{c}_q$, i.e., the symbol is repeated, all M hops are shared and $\varsigma(\mathbf{c}_q, \mathbf{c}_{q+k}) = M$. The contribution of these two symbols to the overall AF at delay k is then M, leading to a higher sidelobe. This problem was addressed in [27], where a method was proposed to deal with symbol repetition by replacing the second copy of the repeated symbol in a pulse with another symbol selected from the discarded set resulting from the dictionary truncation. The replacement symbol is chosen to ensure minimal overlap with its neighbors. However, this approach only addresses the problem of complete symbol overlap, and no attention is paid to other orders of overlap, that is, to cases where $\varsigma(\mathbf{c}_q, \mathbf{c}_{q+k}) = M - n$, for $n = 1, 2, \ldots, M/2$. Yet, these other orders of overlap comprise a large contribution to and result in significant degradation in the AF. The higher the overlap or contribution, the worse the sensing performance. In fact, the impact of the $M - n$ overlaps on AF becomes more pronounced in systems with a larger number of antennas.

Another strategy to improve the AF was to balance the symbols across the pulse [29]. In this approach, the hops within a chip are permuted and distributed evenly across the waveforms to reduce the maximum SLL at delay 1 across the individual AFs. However, this approach does not result in a change in the overall AF.

Therefore, an optimized strategy is required to enhance the sensing performance by tackling the impact of partial overlaps. The aim is to produce significant improvement in the AF, which requires sacrifices in communications resources. In the following section we present a method that trades communications performance for sensing performance to improve the AF at integer delays near the main lobe.

IV. Proposed Minimum overlap approach

To enable the communication performance to be adjusted in order for the sensing to achieve a desirable operation, we propose to selectively exclude some hops from the set of hops available for each chip. This reduces the number of overlapping hops between successive sub-pulses and offers significant improvements in the AF performance at the cost of a reduction of communication bit rate.

Let K be the total number of available hops, $K' \leq K$ be the number of hops allocated for communication in each chip, and $\kappa = K - K'$ be the number of excluded hops. In order to ensure that a symbol does not take up more than half the available hops, we require that $2M \leq K' \leq K$. Let $\mathcal{K} \triangleq \{1, 2, \ldots, K\}$ denote the set of K hops, and define $\mathcal{K}'_q$ as the hops designated for use in the qth chip, such that

$$\mathcal{K}'_q \subseteq \mathcal{K}, \quad \mathcal{K}'_q \triangleq \{\mathcal{K} \setminus \mathcal{E}_q\}, \tag{13}$$

where $\mathcal{E}_q$ is the set of excluded hops in the qth chip with cardinality $|\mathcal{E}_q| = \kappa$. To reduce the probability of symbol overlap and improve the AF value at a delay of 1, we constrain the sets $\mathcal{E}_q$ to satisfy $\mathcal{E}_q \neq \mathcal{E}_{q+1}$. Thus, the hops excluded in a chip are made available for the adjacent chips.

The value of κ determines the number of bits that can be transmitted in a chip. It can be set either to guarantee a minimum bit rate for communication or to keep the maximum SLL of the AF below a certain value. Thus, κ is a trade-off parameter as it offers the sought flexibility in controlling the system performance and its choice should be driven by application-specific priorities. The hops in the set $\mathcal{E}_q$ and overlaps between them play a major role in the AF SLLs. Depending on the bit rate requirements, we can design the excluded subsets such that $\mathcal{E}_q \cap \mathcal{E}_{q+\tau} = \emptyset$, where $\emptyset$ denotes the null set. Thus, using this method, we can also improve the AF at other delays in a relatively easy manner.

The set $\mathcal{K}$ can be partitioned into multiple non-overlapping subsets based on the parameter κ. Let $n_s = \lfloor K/\kappa \rfloor$ be the number of such subsets of excluded hops. The value of n_s also gives the maximum possible number of adjacent chips with non-overlapping sets of excluded hops. Note that $\mathcal{E}_q$ is not unique, and different sets of excluded hops result in different AF profiles. The design of $\mathcal{E}_q$ is treated differently for the two cases: $n_s \geq Q$ (or smaller κ) and $n_s < Q$ (or larger κ). For $n_s \geq Q$, the set $\mathcal{E}_q$ can be designed such that no two chips in a pulse share any common excluded hops. That is,

$$\mathcal{E}_i \cap \mathcal{E}_l = \emptyset, \forall i, l \in [1, Q], \quad i \neq l. \tag{14}$$

For $n_s < Q$, we have that

$$\mathcal{E}_q \cap \mathcal{E}_{q+\tau} = \emptyset, \quad \tau \in [1, n_s]. \tag{15}$$

$\mathcal{E}_q$ may be designed by filling the first n_s chips with non-overlapping hops. Then, for the remaining chips the hops can be chosen from the first n_s chips. This significantly improves the AF value at delays where the excluded hops do not overlap, but leads to higher SLLs at delays where there is a greater overlap between the excluded hops.

For $n_s < Q$, a more prudent approach would be to design the sets $\mathcal{E}_q$ in such a way that the maximum SLL at small integer delays is minimized. As we move farther away from the main lobe, the number of chips that contribute to the AF reduces. We use three integer delays, $\tau = k\Delta_t$ where $k = 1, 2, 3$. Since selecting the optimal set may not be feasible for large values of K and κ, we set the maximum permissible AF SLL at these delays and choose the subset that achieves this.

We now turn our attention to the chip-based symbol dictionary. Given $\mathcal{E}_q$, the dictionary can be designed by choosing M hops from the subset $\mathcal{K}'_q$ containing K' hops. This can be done in $\hat{L} = \binom{K'}{M}$ ways. Let $\mathcal{D}^{(q)}$ denote the symbol dictionary used in the qth chip, defined as $\mathcal{D}^{(q)} \triangleq \{\mathbf{s}_1^{(q)}, \mathbf{s}_2^{(q)}, \ldots, \mathbf{s}_{\hat{L}}^{(q)}\}$ with cardinality $\hat{L}$. The dictionary must be truncated to a power of two to facilitate the communication modality.

The chip-based symbol dictionary is utilized to transmit the communication symbols with $\hat{N}_b = \lfloor \log_2 \hat{L} \rfloor$ representing the number of bits transmitted per chip. A total of $Q\hat{N}_b$ bits are transmitted per pulse. Compared to the FHCS scheme the reduction in bit rate is $N_b - \hat{N}_b$, where $N_b = \lfloor \log_2 \binom{K}{M} \rfloor$.

A. Decoding at Communication Receiver

We assume chip-level synchronization at the communication receiver, which can be achieved through various strategies including pilot symbols. Also, the receiver is assumed to have full knowledge of $\kappa, \mathcal{K}_q, \mathcal{E}_q$ and the chip-based symbol dictionary $\mathcal{D}^q$. The rest of the decoding process is similar to that of the FHCS scheme [21].

V. Results and Discussion

We consider an FH MIMO DFRC system with $BW = 100$ MHz, PRF $=$ 100 kHz, center frequency $f_c = 8$ GHz, and duty cycle of 10%. The pulse duration $T_w = 1$ µs is further divided into $Q = 5$ chips, each having a duration of $\Delta_t = 0.2$ µs. All reported simulations use 10^5 Monte Carlo runs.

First, we compare in Fig. 1 the zero-Doppler cut of the average AF (average of the sum of the AAFs and CAFs) of the proposed scheme with FHCS. In this simulation we use $M = 5$. For the proposed scheme, we put $K = 20$ and $K' = 10$, resulting in $\kappa = 10$ excluded hops in the proposed scheme. For the standard FHCS scheme, we show results for $K = 10$ and $K = 20$. The radar-only AF performance using optimal codes obtained by simulated annealing is also shown for reference. Compared to the FHCS $K = 10$ case, we observe a significant improvement in AF at all delays (including non-integer delays) with a 6 dB gain at delay 1. The enhanced performance can be attributed to the reduction in hop overlaps of the proposed scheme, which uses a separate symbol dictionary in each chip with fewer overlapping symbols. Furthermore, the proposed strategy exhibits a much lower AF at delay 1 compared to the FHCS scheme with $K = 20$, where the improvement is approximately 3.3 dB. The significance of this improvement will be evident from the detection probability simulations presented later.

The improved performance of the proposed scheme is achieved for the same bit rate as the FHCS scheme with $K = 10$ but comes at the expense of a reduced bit rate from 13 bits/chip for FHCS with $K = 20$ to 7 bits/chip for the proposed scheme. Note that all schemes employ the the pulse balancing approach outlined in [29], as it improves the AF given the symbols comprising the pulse.

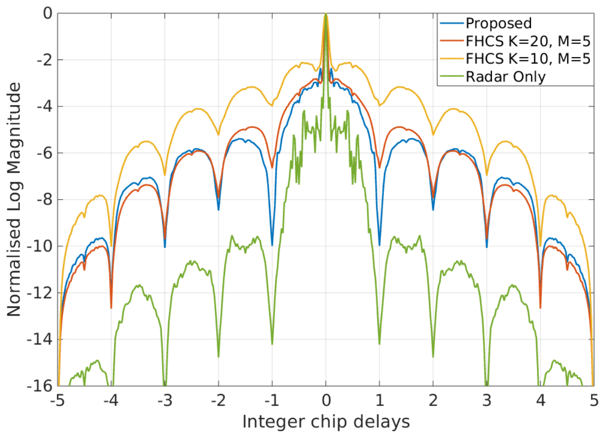

Fig. 1. Zero-Doppler AF curves of chip-based and FHCS schemes for $M = 5, Q = 5, K = 20$ and $K' = 10$.

Fig. 2 shows the variation of average AF value at delay 1 (left y-axis) and the bit rate of the proposed scheme (right y-axis) with the trade-off parameter κ. For $K = 20, M = 5$ and $Q = 5$, κ is varied from 2 to 10 with a step of 2. For each κ, we also compute the FHCS AF with K setting to the respective K'. The lowest achievable AF value at delay 1 for the FHCS with $K = 20$ and $M = 5$ is -6.67 dB and is depicted as a horizontal reference line. As κ increases, the proposed scheme exhibits an improved AF value at delay 1 by trading off an increased number of bits.

Finally, we compare the detection performance of the proposed approach for $\kappa = 2$ and 10 to that of the FHCS scheme and radar-only waveform. In the simulation, we evaluate the probability of detection, P_{d}, for a probability of false alarm of 0.001. We fix the clutter-to-noise ratio to 15 dB and insert a target at azimuth angle 20°. For all schemes, we set $K = 20$, $M = 5$ and $Q = 5$. In Fig. 3 we show curves of the P_{d} versus signal to noise ratio (SNR). We see that the proposed scheme achieves a better detection performance than the FHCS scheme for both values of κ. As κ increases, the P_{d} of the proposed scheme improves approaching the radar-only scenario. This is

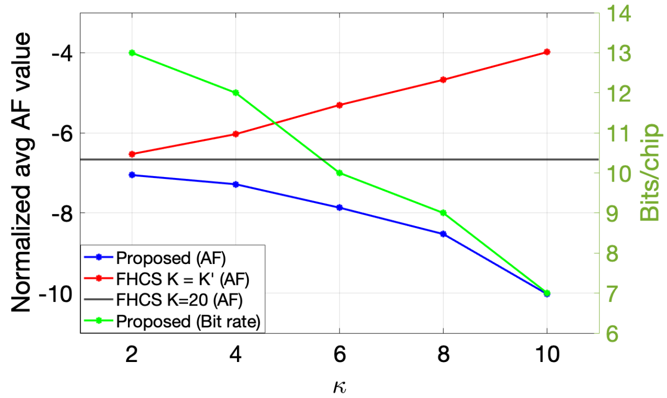

Fig. 2. AF value at delay 1 and bit rate versus κ for $K = 20, M = 5, Q = 5$.

due to the decrease in the number of overlapping hops. This demonstrates that the proposed scheme allows the performance of the communications and sensing modalities to be traded off against each other.

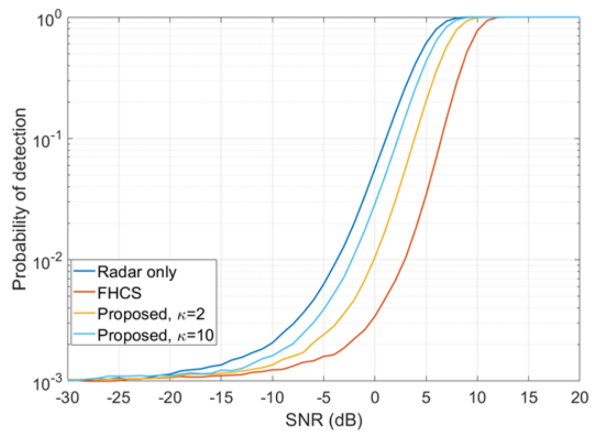

Fig. 3. Probability of detection versus SNR for $K = 20, M = 5, Q = 5$.

VI. Conclusion

The FHCS scheme exhibits poor sensing performance, offering no means to control this aspect of its operation. In this paper, we addressed this problem, proposing an approach that enables communications performance to be traded for improved sensing. We achieved this by excluding some hops from each chip, allowing us different symbol dictionaries to be used in different chips. By trading off bit rate, we can thus significantly enhance the AF performance at delays closer to the main lobe, which also improves the probability of detection. We demonstrated through simulations that the proposed scheme enhances the sensing performance compared to the FHCS scheme. Our approach can be further extended to permit dynamic dictionary design, which is the subject of future research.

REFERENCES

[1] A. Hassanien, M. G. Amin, E. Aboutanios, and B. Himed, "Dual-function radar communication systems: A solution to the spectrum congestion problem," *IEEE Signal Processing Magazine*, vol. 36, no. 5, pp. 115–126, 2019.

[2] L. Zheng, M. Lops, Y. C. Eldar, and X. Wang, "Radar and communication coexistence: An overview: A review of recent methods," *IEEE Signal Processing Magazine*, vol. 36, no. 5, pp. 85–99, 2019.

[3] K. V. Mishra, M. Bhavani Shankar, V. Koivunen, B. Ottersten, and S. A. Vorobyov, "Toward millimeter-wave joint radar communications: A signal processing perspective," *IEEE Signal Processing Magazine*, vol. 36, no. 5, pp. 100–114, 2019.

[4] F. Liu, C. Masouros, A. P. Petropulu, H. Griffiths, and L. Hanzo, "Joint radar and communication design: Applications, state-of-the-art, and the road ahead," *IEEE Transactions on Communications*, vol. 68, no. 6, pp. 3834–3862, 2020.

[5] A. Ahmed, E. Aboutanios, and Y. D. Zhang, "Sensing-centric ISAC signal processing," in *Integrated Sensing and Communications*, F. Liu, Y. C. Eldar, and C. Masouros, Eds. Springer, 2023.

[6] A. Hanif, S. Ahmed, T. Y. Al-Naffouri, and M.-S. Alouin, "Exploring the synergy: A review of dual-functional radar communication systems," *arXiv preprint arXiv:2401.00469*, 2023.

[7] A. Hassanien, M. G. Amin, Y. D. Zhang, and F. Ahmad, "Dual-function radar-communications: Information embedding using sidelobe control and waveform diversity," *IEEE Transactions on Signal Processing*, vol. 64, no. 8, pp. 2168–2181, 2015.

[8] J. Euziere, R. Guinvarc'h, M. Lesturgie, B. Uguen, and R. Gillard, "Dual function radar communication time-modulated array," in *2014 International Radar Conference*, 2014, pp. 1–4.

[9] A. Hassanien, M. G. Amin, Y. D. Zhang, and F. Ahmad, "Phase-modulation based dual-function radar-communications," *IET Radar, Sonar & Navigation*, vol. 10, no. 8, pp. 1411–1421, 2016.

[10] J. Li and P. Stoica, Eds., *MIMO Radar Signal Processing*. Wiley, 2009.

[11] A. Hassanien, M. G. Amin, Y. D. Zhang, and B. Himed, "A dual-function MIMO radar-communications system using PSK modulation," in *2016 24th European Signal Processing Conference (EUSIPCO)*, 2016, pp. 1613–1617.

[12] E. BouDaher, A. Hassanien, E. Aboutanios, and M. G. Amin, "Towards a dual-function MIMO radar-communication system," in *2016 IEEE Radar Conference (RadarConf)*, 2016, pp. 1–6.

[13] F. Liu and C. Masouros, "Hybrid beamforming with sub-arrayed mimo radar: Enabling joint sensing and communication at mmwave band," in *ICASSP 2019-2019 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*. IEEE, 2019, pp. 7770–7774.

[14] R. Liu, M. Li, Q. Liu, and A. L. Swindlehurst, "Dual-functional radar-communication waveform design: A symbol-level precoding approach," *IEEE Journal of Selected Topics in Signal Processing*, vol. 15, no. 6, pp. 1316–1331, 2021.

[15] S. Zhu, F. Xi, S. Chen, and A. Nehorai, "A low-complexity mimo dual function radar communication system via one-bit sampling," in *ICASSP 2021-2021 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*. IEEE, 2021, pp. 8223–8227.

[16] A. Hassanien, B. Himed, and B. D. Rigling, "A dual-function MIMO radar-communications system using frequency-hopping waveforms," in *2017 IEEE Radar Conference (RadarConf)*, 2017, pp. 1721–1725.

[17] W. Baxter, E. Aboutanios, and A. Hassanien, "Dual-function MIMO radar-communications via frequency-hopping code selection," in *2018 52nd Asilomar Conference on Signals, Systems, and Computers*, 2018, pp. 1126–1130.

[18] K. Wu, J. A. Zhang, X. Huang, Y. J. Guo, and R. W. Heath, "Waveform design and accurate channel estimation for frequency-hopping MIMO radar-based communications," *IEEE Transactions on Communications*, vol. 69, no. 2, pp. 1244–1258, 2021.

[19] K. Wu, J. A. Zhang, X. Huang, and Y. J. Guo, "Frequency-hopping MIMO radar-based communications: An overview," *IEEE Aerospace and Electronic Systems Magazine*, vol. 37, no. 4, pp. 42–54, 2021.

[20] J. Liu, K. Wu, T. Su, and J. A. Zhang, "Practical frequency-hopping mimo joint radar communications: Design and experiment," *Digital Communications and Networks*, vol. 10, no. 6, pp. 1904–1914, 2024.

[21] W. Baxter, E. Aboutanios, and A. Hassanien, "Joint radar and communications for frequency-hopped MIMO systems," *IEEE Transactions on Signal Processing*, vol. 70, pp. 729–742, 2022.

[22] Y. Xiong, F. Dong, and F. Liu, "Information-theoretic limits of integrated sensing and communications," in *Information-Theoretic Radar Signal Processing*, Y. Gu and Y. D. Zhang, Eds. Wiley-IEEE Press, 2025.

[23] G. San Antonio, D. R. Fuhrmann, and F. C. Robey, "MIMO radar ambiguity functions," *IEEE Journal of Selected Topics in Signal Processing*, vol. 1, no. 1, pp. 167–177, 2007.

[24] M. I. Skolnik, *Introduction to Radar Systems, Third Edition*. McGraw-Hill, 2002.

[25] I. P. Eedara, A. Hassanien, M. G. Amin, and B. D. Rigling, "Ambiguity function analysis for dual-function radar communications using PSK signaling," in *2018 52nd Asilomar Conference on Signals, Systems, and Computers*, 2018, pp. 900–904.

[26] I. P. Eedara, A. Hassanien, and M. G. Amin, "Performance analysis of dual-function multiple-input multiple-output radar-communications using frequency hopping waveforms and phase shift keying signalling," *IET Radar, Sonar & Navigation*, vol. 15, no. 4, pp. 402–418, 2021.

[27] W. Baxter, E. Aboutanios, and A. Hassanien, "Ambiguity function analysis of the frequency-hopped code selection scheme," in *2023 IEEE International Radar Conference (RADAR)*, 2023, pp. 1–6.

[28] W. Baxter, H. Nosrati, and E. Aboutanios, "A study on the performance of symbol dictionary selection for the frequency hopped DFRC scheme," in *2020 IEEE Radar Conference (RadarConf)*, 2020, pp. 1–6.

[29] E. Aboutanios, W. Baxter, and Y. D. Zhang, "Improved implementation of the frequency hopped code selection DFRC scheme," in *2023 IEEE Radar Conference (RadarConf)*, 2023, pp. 1–6.

[30] T. Derham, S. Doughty, C. Baker, and K. Woodbridge, "Ambiguity functions for spatially coherent and incoherent multistatic radar," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 46, no. 1, pp. 230–245, 2010.