

Distributed Radar Network for Real-Time Tracking of Bullet Trajectory

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ABSTRACT

Gunshot detection, sniper localization, and bullet trajectory prediction are of significant importance in military and homeland security applications. While the majority of existing work is based on acoustic and electro-optical sensors, this paper develops a framework of networked radar systems that uses distributed radar sensor networks to achieve the abovementioned objectives. The use of radio frequency radar systems allows the achievement of sub-time-of-flight tracking response, enabling to respond before the bullet reaches its target and, as such, effectively leading to the reduction of injuries and casualties in military and homeland security operations. The focus of this paper is to examine the MIMO radar concept with concurrent transmission of low-correlation waveforms from multiple radar sets to ensure wide surveillance coverage and maintain a high waveform repetition frequency for long coherent time interval required to achieve return signal concentration.

Keywords: MIMO radar, waveform design, sensor network, gunshot location system, bullet trajectory tracking.

1. INTRODUCTION

Gunshot detection, sniper localization, and bullet trajectory prediction are of significant importance in military and homeland security applications. The majority of existing work is based on acoustic sensors (see for example^{1,2}). One of the example is the Boomerang acoustic shooter detection system developed by BBN. These techniques are useful for gunshot detection and sniper localization, but they fail to predict the bullet trajectory because of, among other reasons, the slow propagation velocity of acoustic signals. An alternative solution is to use optical or electro-optical systems that can detect either the physical phenomenon of the muzzle flash of a bullet being fired or the heat caused by the friction of the bullet as it moves through the air. Rapid Target Acquisition and Tracking System (RTATS) developed by Trex Enterprises is a small-aperture optical system designed to handle multiple rocket, mortar, and artillery targets plus provide three-dimensional tracking data to return fire weapons systems. Such systems require that they have a clear line-of-sight (LOS) to the weapon being fired or the projectile while it is in motion. Also, these systems may be defeated by specialized flash suppressors.

This paper develops a framework that uses distributed sensor networks to achieve the three abovementioned objectives. we use radio frequency (RF) sensors in our work. The key advantage of using RF signals allows the achievement of sub-time-of-flight tracking response, enabling to respond before the bullet reaches its target and, as such, effectively leading to the reduction of injuries and casualties in military and homeland security operations as a result of providing individual soldiers with the sub-time-of-flight tracking capability for rapidly determining the location of threats. The use of multi-sensor radar for bullet shells has been considered in^{3,4} with the use of Doppler signatures and dual-frequency signaling.

The major challenges for the detection of tracking of bullet trajectory, on the other hand, lies in the limited range of observation because of the significant propagation attenuation and the small radar cross section (RCS) of the bullet. Depending on the sensing and processing radar technology to be used in the system, the radar may only observe the information of Doppler, range, or both. Some radar systems may provide the directional

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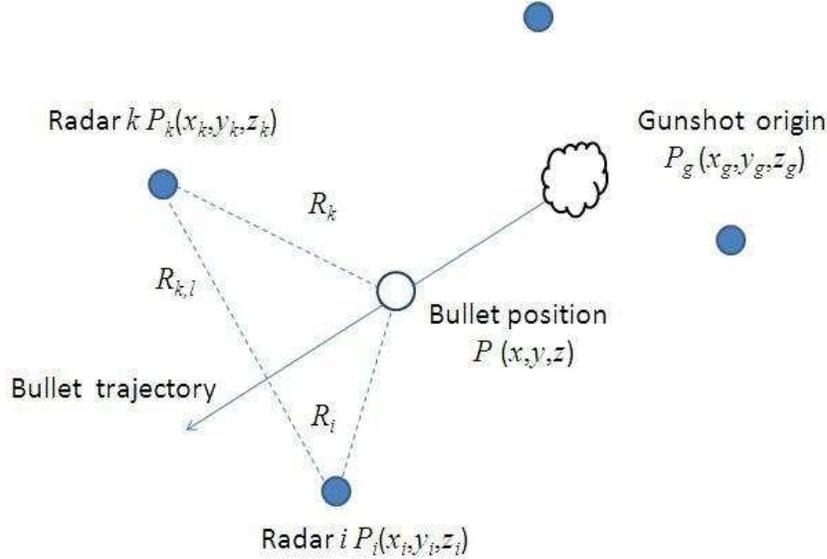


Figure 1. Illustration of a bullet tracking scenario.

information of the target. All these pieces of information are not necessarily enough to uniquely localize the bullet. To solve these problems, it is desirable to construct a sensor network that is distributed over the area of interest. As such, the entire area of interest will under immediate surveillance for quick response. Mono- or bi-static radar processing at multiple radar sites also allows multi-view information collection for improved detection and tracking. Once the range information of the bullet with respect to at least three radar sets is obtained, the location of the bullet can be identified at each time instant through trilateration. Optionally, Doppler information can also be incorporated to improve the bullet trajectory tracking over time. Exploiting simple integral of Doppler information as well as Kalman filtering techniques has been considered in.^{5,6}

A commonly used approach for transmitting sensing signals from multiple radar sites is to sequentially schedule their transmission. While this approach is relatively simple, it is not necessarily effective because of the reduction in the waveform repetition frequency (WRF). Therefore, collecting more data samples within the limited time will result in a higher Fourier transform gain. As a result, a high WRF is required. This problem is solved by using multiple low-correlation waveforms, for example, linear frequency-modulated (LFM) chirp signals with different chirp rates.

This paper is organized as follows. Section 2 introduces the system and signal models. The bullet localization techniques are discussed in Section 3. Section 4 provides simulation results.

2. SYSTEM AND SIGNAL MODELS

2.1 System Model

Figure 1 illustrates the problem of gunshot location and bullet tracking. Several radar sets are positioned around the gunshot origin as well as the trajectory of the bullet. The data observed at each radar set can be processed individually or, more effectively, forward summarized information data, through a real-time wireless or wired connection, to the fusion center for centralized data processing.

Consider that two fixed radar sets i and k , which are respectively located at point P_i with a coordinate of (x_i, y_i, z_i) and at point P_k with a coordinate of (x_k, y_k, z_k) . For an arbitrary bullet position P located at $(x(t), y(t), z(t))$ along its trajectory at a time instant t , the distance between the bullet at the two radar sets is denoted, respectively, as R_i and R_k . The distance between radars i and k is denoted as $R_{i,k}$, which can be determined from the known positions of the two radars.

Let us first assume that only radar set i transmits a signal which is received at all the radar sets in the network, including radar set i itself and radar set k . As a result, radar set i operates in a mono-static mode and

observes the round-trip range $2R_i$ as well as the Doppler frequency $-2\dot{R}_i f/c$, where $\dot{R}_i = dR_i/dt$, f is the carrier frequency, and c is the velocity of RF wave propagation. On the other hand, radar set k operates in a bi-static mode and observes the sum ranges of $R_i + R_k$ and the Doppler frequency $-(\dot{R}_i + \dot{R}_k)f/c$. The delay of the signal reflected from the bullet represents the sum range $R_i + R_k$. From the time delay between the radar return signal and the direct path, on the other hand, $R_i + R_k - R_{i,k}$ can be obtained. Using the known information of $R_{i,k}$, the sum range $R_i + R_k$ can be obtained at radar k by correlating the direct path from radar i and the return signal from the bullet. Once the range information of the bullet with respect to at least three radar sets is obtained, the location of the bullet can be identified at each time instant through trilateration.

As we discussed above, the radar range is limited due to the small RCS of the bullet. To ensure a wide surveillance coverage, it is desirable that all the radar sets transmit their respective waveforms so that reliable data observations can be obtained by using the waveforms transmitted from those radar sets that are close to the bullet. A commonly used approach for transmitting sensing waveforms from multiple radar sets is to sequentially schedule their transmission. While this approach is relatively simple in terms of signal detection, it is not necessarily effective because the waveform repetition frequency is proportionally reduced by the sequential transmission scheduling. To collect enough data samples within the limited time of observation, a high WRF is required. This problem is solved by using multiple low-correlation waveforms, for example, LFM chirp signals with different chirp rates.

The use of multiple low-correlation waveforms is an area that attracts wide attention in the MIMO radar community.⁷ Among them, chirp waveforms with rising and falling chirp rates are used for a two-radar system.^{8,9} Only the matched waveform resolves the target in the range-Doppler map, whereas other waveforms resolve the Doppler but not in the range domain. As such, a CLEAN technique can be used to mitigate multi-radar interference.^{8,9}

In this paper, a networked radar system with multiple radar sets are considered, and each radar uses a respective waveform which has a low correlation with the waveforms transmitted from other radars. When N radar sets are used, the MIMO radar processing provides a total number of N^2 data sets at each time instant. Among them, some may not necessarily provide reliable range-Doppler information of the bullet under tracking. Each radar system may receive and process the N data sets, and only those with reliable range-Doppler estimates are used in the bullet localization and tracking. The range only information of the bullet observed at a radar corresponding to the waveforms transmitted from multiple radar sets allows trilateration for the localization of the bullet at each time instant, while the Doppler information can also be fused with the range data to improve the localization and trajectory tracking.^{5,6} When received signals observed at all the N radar sets are used, each radar set can forward the processed data, which requires very small traffic size, to the fusion center for improved bullet localization and tracking performance.

The detailed signal processing techniques are described in the following section.

2.2 Signal Model

Consider that a frequency-modulated continuous-wave (FM/CW) waveform consisting of a coherent series of LFM chirps is transmitted from each radar set in order to determine the range of bullet and its Doppler information. The m th waveform transmitted from radar set i , $i = 1, \dots, N$, is an LFM chirp signal of the form

$$v_{i,0}(t) = \begin{cases} \exp(j\pi\tilde{\kappa}_i B_i f_{r,i} t^2), & 0 \leq t < T_{r,i}, \\ 0, & \text{otherwise,} \end{cases} \quad (1)$$

where $T_{r,i}$ and $f_{r,i} = 1/T_{r,i}$ are respectively the waveform repetition interval (WRI) and WRF, and B_i is the bandwidth of the chirp. In addition, $\tilde{\kappa}_i$ takes value of either 1 and -1 , depending on the sign of chirp rate. That is, $\tilde{\kappa}_i = 1$ for a chirp with a raising frequency whereas $\tilde{\kappa}_i = -1$ for a chirp with a falling frequency. The radar signal transmitted from radar set i , $v_i(t)$, is a series of M LFM waveforms, i.e.,

$$v_i(t) = \alpha_0 \exp(j\omega_0 t) \sum_{m=0}^{\infty} v_{i,0}(t - mT_r), \quad (2)$$

where α_i is a complex scaler representing the transmitted amplitude and phase, β_i is the chirp rate, $\omega_0 = 2\pi f_0$ is the radar operation frequency.

For a bullet s with a distance of R_i to radar i and R_k to radar k , the noise-free signal received at radar set k corresponding to the m th transmitted waveform is

$$u_{k,i}(t) = \tilde{\alpha}_{k,i} \exp \left[j(\omega_0 + \omega_i + \omega_k) \left(t - \frac{R_i}{c} - \frac{R_k}{c} \right) \right] \sum_{m=0}^{\infty} v_{i,0} \left(t - \frac{R_i}{c} - \frac{R_k}{c} - mT_{r,i} \right), \quad (3)$$

where $\tilde{\alpha}_{k,i}$ represents the received amplitude and phase corresponding to transmit radar i and receive radar k , c is the velocity of light, $\omega_i = 2\pi f_0 \dot{R}_i/c$ and $\omega_k = 2\pi f_0 \dot{R}_k/c$ are, respectively, the Doppler frequency shift of the bullet in the direction of radar sets i and k . Note that, when $i = k$, the above received signal corresponds to the mono-static operation of radar i , whereas it is a bi-static operation when $i \neq k$.

When N radar sets transmit their respective waveforms, the signal received at the k th radar set is the superposition of those corresponding to different transmit radars, and is expressed as

$$\begin{aligned} u_k(t) &= \sum_{i=1}^N u_{k,i}(t) + \eta(t) \\ &= \sum_{i=1}^N \tilde{\alpha}_{k,i} \exp \left[j(\omega_0 + \omega_i + \omega_k) \left(t - \frac{R_i}{c} - \frac{R_k}{c} \right) \right] \sum_{m=0}^{\infty} v_{i,0} \left(t - \frac{R_i}{c} - \frac{R_k}{c} - mT_{r,i} \right) + \eta_k(t), \end{aligned} \quad (4)$$

where $\eta_k(t)$ represents the additive noise observed at radar set k .

For the effectiveness of the operation and the simplicity of representation, we assume that different radar sets use the same bandwidth, i.e., $B_1 = \dots = B_N = B$. We also assume that the radar sets either have the same WRI, or the longest WRI of some radars is the integer multiple of the shorter WRI's that the other radars use. In this way, we define $T_r = \max_{i=1, \dots, N} T_{r,i}$ and thus $T_{r,i} = T_r/|\kappa_i|$, where κ_i is a nonzero integer and takes the same sign as $\tilde{\kappa}_i$. Similarly, we define $f_r = \min_{i=1, \dots, N} f_{r,i}$ and $f_{r,i} = |\kappa_i|f_r$. Note that, however, the maximum WRF $f_{r,\max} = \max_{i=1, \dots, N} f_{r,i}$ has to properly configured so that the range ambiguity requirement is met.

In this paper, the effect of clutter is not considered. The clutter can be effectively removed through background subtraction. That is, in the absence of the target (bullet), we can measure the background signal observation that is purely due to clutter. The effect of noise in clutter estimation is considered negligible when sufficient large number of samples are used for averaging.

In the process of dechirpping, the received signal is mixed with a delayed version of the signal transmitted from one of the radar sets. When the waveform of radar set g is used to dechirp the received waveform, we have

$$w_{g,k}(t) = u_k(t)v_g^*(t) \quad (5)$$

where “*” denotes complex conjugate. Passing $w(t)$ through a low pass filter (LPF) to remove the $2f_0$ component results in a signal, $q(t)$, which is dechirped with respect to the waveform transmitted from radar set g . The LPF output is expressed as

$$\begin{aligned} q_{g,k}(t) &= \alpha_{k,g} \exp[j(\omega_g + \omega_k)t] \sum_{m=0}^{\infty} \exp \left[-j2\pi B\kappa_g f_r \left(\frac{R_g}{c} + \frac{R_k}{c} \right) (t - mT_r) \right] \\ &+ \sum_{\substack{i=1 \\ i \neq g}}^N \alpha_{k,i} \exp[j(\omega_i + \omega_k)t] \sum_{m=0}^{\infty} \exp \left[-j2\pi B\kappa_i f_r \left(\frac{R_i}{c} + \frac{R_k}{c} \right) (t - mT_r) + j\pi B(\kappa_i - \kappa_k) f_r (t - mT_r)^2 \right] \\ &+ \xi_k(t) \end{aligned} \quad (6)$$

where the constant phase and amplitude terms are lumped into $\alpha_{k,i}$ and where $\xi_k(t)$ is the low pass filtered noise. The first term at the right-hand side (RHS) of the above equation is the dechirped target return corresponding to the waveform transmitted from radar g , whereas the second term represents the cross-radar interference.

The above results are typically sampled and further processed by using a two-dimensional (2-D) Fourier transform to get the range-Doppler image results. To achieve a high signal concentration, it is desirable to use a large number of samples in both the fast snapshot and the number of repetitive waveforms. As a result, the first term at the RHS of the equation (6) will result in a resolved solution in the joint range-Doppler domain, thus yield a single peak around the true position in the range-Doppler image. On the other hand, the second term at the RHS of equation (6) resolves only in the Doppler domain but fails to resolve in the range domain. As a result, each of them appears as a line the range-Doppler image.

Note that, in the bi-static operation mode, the exact time that the waveform was transmitted from a spatially separately radar may not be precisely known. However, because the direct path from the transmit radar is expected to be strong, the transmit timing can be estimated from the time delay of the direct path and the distance between the two radars, which can be obtained from the known radar positions.

3. LOCALIZATION TECHNIQUES

From the range-Doppler images obtained at radar $i, i = 1, \dots, N$, located at $P_i(x_i, y_i, z_i)$, we can estimate the auto-range R_i and cross-range $R_i + R_k, k = 1, \dots, N, k \neq i$, respectively. In this paper, we consider the use of range information for the bullet localization. As we discussed earlier, Doppler information can also be incorporated to improve the bullet trajectory tracking over time. Exploiting simple integral of Doppler information as well as Kalman filtering techniques has been considered in.^{5,6} The consideration of these techniques, however, is out of the scope of this paper and will be considered in other opportunities.

When a radar processes the return signal respectively using the N different waveforms, a full set of the N range estimates, $\hat{R}_i, i = 1, \dots, N$, is obtained at each radar receiver. When the observation at all the N radar sets are used, N copies of the full range estimation sets can be obtained. With the use of these range estimates, the bullet position, denoted as $P(x, y, z)$, can be localized using the trilateration/multilateration method (see, for example,¹⁰). As a general rule, to unambiguously localize a target in an Q -dimensional space, range information is required from at least $Q+1$ reference points. However, in some cases, only Q reference points are required if the range ambiguity is not a problem. For example, the bilateration method that uses two reference points yields two intersections in a two-dimensional (2-D) plane. In the following, we discuss several localization techniques, which differ in the use manners of the range estimates mentioned above.

3.1 Auto-range-based Multilateration

This approach assumes that all the radars operate in mono-static operation. As a result, there is no cross-range estimation for $\hat{R}_{k,i} = R_i + R_k$. The bullet is localized using the multilateration technique based on the auto-range estimations, $\hat{R}_i, i = 1, \dots, N$. Based on the known coordinates of the radars and the estimates of the auto-ranges \hat{R}_i , the following range equations can be established

$$(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2 = \hat{R}_i^2, i = 1, \dots, N. \quad (7)$$

When $N > 4$, this localization problem becomes over-determined. Consequently, the least-square solution of the bullet location is obtained by solving the following over-determined problem

$$\mathbf{A}\mathbf{p} = \mathbf{B}, \quad (8)$$

where

$$\mathbf{A} = \begin{bmatrix} x_1 - x_2 & y_1 - y_2 & z_1 - z_2 \\ \vdots & \vdots & \vdots \\ x_{N-1} - x_N & y_{N-1} - y_N & z_{N-1} - z_N \end{bmatrix}$$

is a $(N-1) \times 3$ matrix, $\mathbf{p} = [x, y, z]^T$ is an $(N-1) \times 1$ vector, where $(\cdot)^T$ denotes the transpose of a matrix or a vector, and

$$\mathbf{B} = \begin{bmatrix} \hat{R}_2^2 - \hat{R}_1^2 + x_1^2 - x_2^2 + y_1^2 - y_2^2 + z_1^2 - z_2^2 \\ \vdots \\ \hat{R}_N^2 - \hat{R}_{N-1}^2 + x_{N-1}^2 - x_N^2 + y_{N-1}^2 - y_N^2 + z_{N-1}^2 - z_N^2 \end{bmatrix}$$

is an $(N - 1) \times 1$ vector. A convenient and effective method to solving such an over-determined problem is the least-squares approach, which provides a solution to equation (8) by

$$\mathbf{p} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{B}. \quad (9)$$

It is worthy noting that when some values of $\widehat{R}_i, i = 1, \dots, N$, become unreliable, they should be excluded from equations (7) and (8) so as to improve the localization accuracy. Moreover, if the number of equations in (7) is less than 4, the bullet could not be uniquely localized unless the location ambiguity is not a problem.

3.2 Auto- and Cross-range-based Multilateration

When the cross-range estimates $\widehat{R}_{k,i} = \widehat{R}_i + \widehat{R}_k$ can be reliably estimated, the bullet position can be localized using the auto-range estimates, \widehat{R}_i , and cross-range estimates, $\widehat{R}_{k,i}$, for $i = 1, \dots, N$ and $k = 1, \dots, N$. In general, there are two steps for this kind of localization technique. Specifically, each radar individually localizes the bullet with the help of other radars, and the localization is then improved by fusing all the estimation results. Thus, it can localize the bullet even some of the radar returns do not provide reliable range estimations. This differs from the ‘‘auto-range-based multilateration’’ described previously, where all the radars are required to provide reliable range estimate.

With the use of cross-range estimates $\widehat{R}_{k,i}$ and auto-range \widehat{R}_i , the range between the bullet and the k th radar can be estimated by the radar i as

$$\widehat{R}_k^{(i)} = \widehat{R}_{k,i} - \widehat{R}_i, \quad k = 1, \dots, N, k \neq i. \quad (10)$$

As such, the bullet can be localized by radar i using a similar multilateration method and equation (8) is updated as

$$\mathbf{A} \mathbf{p}_i = \mathbf{B}_i, \quad (11)$$

where

$$\mathbf{B}_i = \begin{bmatrix} \widehat{R}_2^{(i)2} - \widehat{R}_1^{(i)2} + x_1^2 - x_2^2 + y_1^2 - y_2^2 + z_1^2 - z_2^2 \\ \vdots \\ \widehat{R}_N^{(i)2} - \widehat{R}_{N-1}^{(i)2} + x_{N-1}^2 - x_N^2 + y_{N-1}^2 - y_N^2 + z_{N-1}^2 - z_N^2 \end{bmatrix},$$

is an $(N - 1) \times 1$ vector, and \mathbf{p}_i is the estimate of \mathbf{p} at radar i . Consequently, the least-square solution of equation (11) is given by

$$\mathbf{p}_i = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{B}_i, \quad i = 1, \dots, N. \quad (12)$$

Two different methods for the fusion of the obtained bullet position estimates are described below.

(a) Arithmetic Average

This is a direct method, where the bullet location is improved by arithmetically averaging the location estimates obtained at each radar. In this case, the bullet’s location is written as

$$\mathbf{p} = \frac{1}{N} \sum_{i=1}^N \mathbf{p}_i = \frac{1}{N} (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \sum_{i=1}^N \mathbf{B}_i. \quad (13)$$

(b) Weighted Average

This method uses a weighted average with the use of proper weights. In this case, the bullet’s location can be expressed as

$$\mathbf{p} = \sum_{i=1}^N w_i \mathbf{p}_i = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \sum_{i=1}^N w_i \mathbf{B}_i. \quad (14)$$

Apparently, the arithmetic average is a special case by setting $w_i = 1, i = 1, \dots, N$.

In this fusion method, the weights $w_i, i = 1, \dots, N$, corresponding to the estimates with higher reliability are desirable to be larger. Generally, the location estimation under higher target signal level (i.e., smaller range) is more reliable, and thus high weights should be allocated to small values of \widehat{R}_i . For example, the i th weight can be chosen as

$$w_i = \frac{1/\widehat{R}_i}{\sum_{k=1}^N 1/\widehat{R}_k}. \quad (15)$$

Note that when some values of $\mathbf{p}_i, i = 1, \dots, N$, become unreliable, they should be excluded during the fusion operations to improve the localization accuracy.

4. SIMULATION EXAMPLE

To demonstrate the effectiveness of the proposed scheme for real-time tracking of bullet trajectory, we establish a simulation platform.

4.1 Assumptions

We consider a networked radar system consisting of four radars, operating at the carrier frequency 2 GHz. The sweep frequency bandwidth is 50 MHz, and sampling rate is set to 100 MHz. Assume that Radar 1 and Radar 2 operate at a WRF of 125 KHz (i.e., there are 800 samples per sweep). 1024 sweeps are used to obtain the range-Doppler map, yielding a coherent integration time (CIT) of 8.192 ms. They share the same chirp rate, but Radar 1's chirp waveform has a raising chirp rate whereas Radar 2's chirp waveform has a falling chirp rate. Further, Radar 3 and Radar 4 are assumed to use a doubled WRF of 250 KHz (i.e., there are 400 samples per sweep). The CIT remains to be 8.192 ms, which corresponds to 2048 sweeps of the waveforms used by Radar 3 and Radar 4. Similarly, Radar 3 and 4 have the same chirp rate, but Radar 3 has a positive chirp rate whereas Radar 4 has a negative chirp rate. These four radars are, respectively, placed at coordinates $P_1(-120, 25)$, $P_2(-240, 5)$, $P_3(-100, -100)$, and $P_4(0, 0)$, which are marked in Fig. 3 as triangles. The bullet flies to southeast in the direction of -20° with respect the x axis. The initial bullet location at $t = 0$ is $P(-200, 20)$, and the initial velocity of the bullet is 830 m/s. Seven observation results, separated by 50 ms, are selected to be depicted. The transmit power of reach radar is fixed at 10 Watts.

4.2 Simulation Results

In Fig. 2, we depict 16 range-Doppler images of the bullet, located at $P(-161, -34)$, as the combined results of four receive radars and four dechipping waveforms (DW). Each peak is highlighted by a circle. The cross-radar interference is buried under noise in the results. When the peak is reliable, the range information can be accurately extracted within the range resolution accuracy. Based on the range information, the bullet is localized using the localization techniques described in Section 3. The located positions are illustrated in Fig. 3 for the three different methods. Legend "Loc-method I" denotes the auto-range-based multilateration technique, whereas "Loc-method IIa" and "Loc-method IIb" respectively denote the auto- and cross-range-based multilateration technique using the "arithmetic average" and "weighted average" techniques. These results show that the use of the proposed schemes, in general, provide reliable locating and tracking performance.

The absolute locating errors of various localization methods are plotted in Fig. 4. In all the results depicted in this figure, the location errors are smaller than 3 meters. Comparing the "Loc-method IIa" and "Loc-method IIb", it is observed that location error is reduced by using the weighted average method. As mentioned in Section 3, the "Loc-method I" method requires at least three radars which reliably estimate the auto-range, whereas the other two methods do not have such a restrict. As a result, the "Loc-method I" method fails to localize the bullet in some harsh cases. For example, when the bullet is at the 7th observation point, the "Loc-method I" method cannot locate the bullet since Radar 1 and Radar 2 do not reliably estimate individual auto-range. On the other hand, the other two methods can still locate the bullet, because some of other combined paths provide reliable range estimation performance.

5. CONCLUSION

We have considered the networked radar framework for the detection and localization of a bullet. By using a radar network, reliable detection and localization is achieved. In particular, the combined use of mono- and bi-static modes yields much richer information to enhance the quality and reliability of bullet localization. While this paper only used the range information, the incorporation of Doppler information is considered as future work and is expected to further improve the localization performance.

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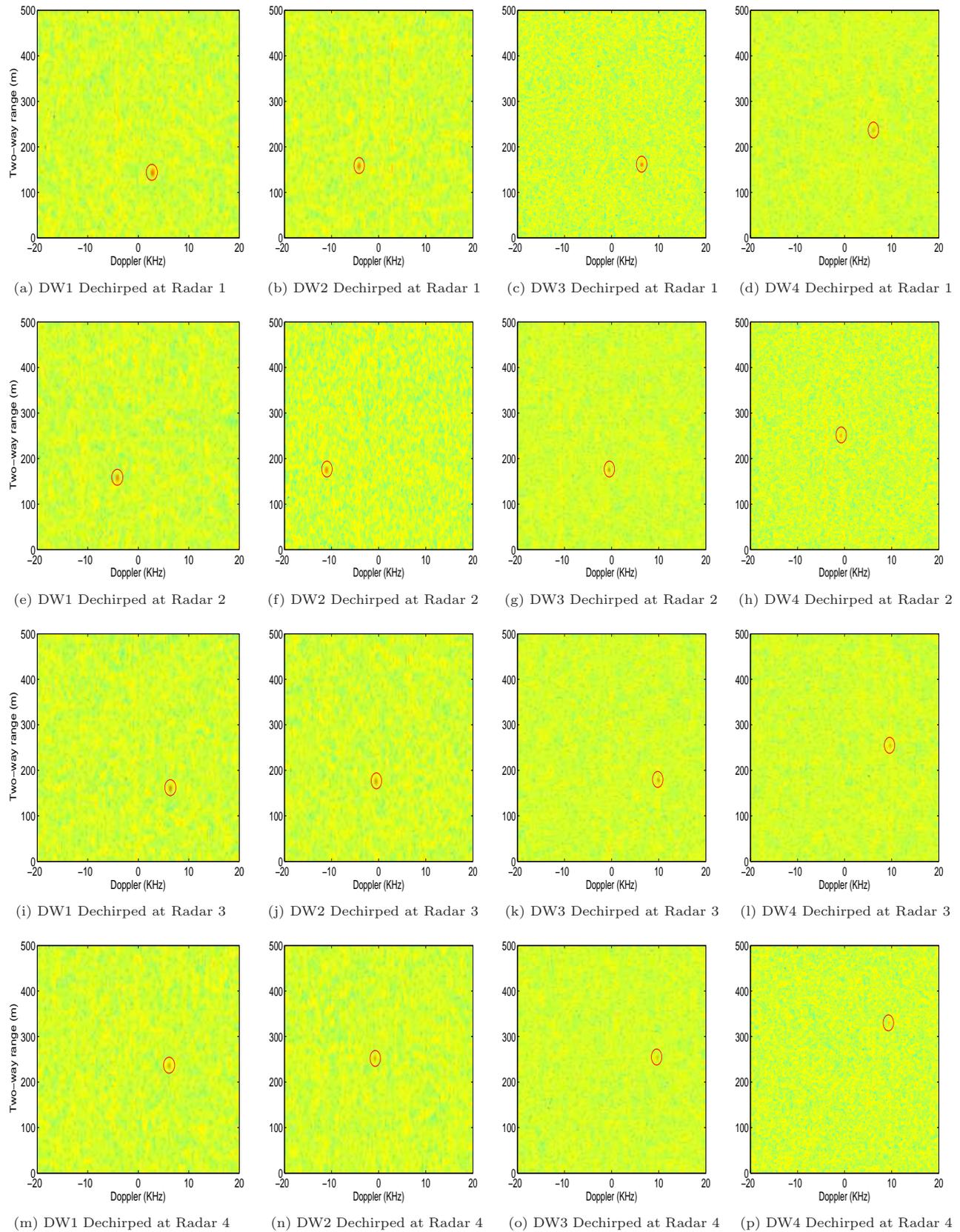


Figure 2. Range-Doppler images of the four DWs in the second observation individually dechirped at the four Radars.

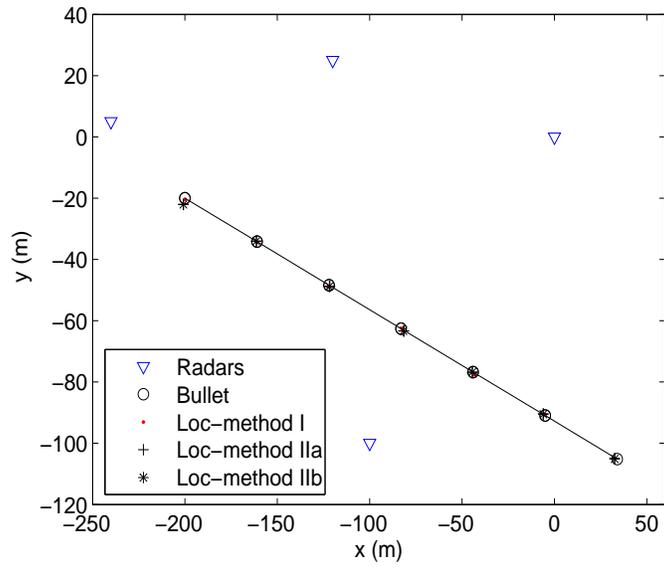


Figure 3. Locating and tracking the bullet using three localization methods.

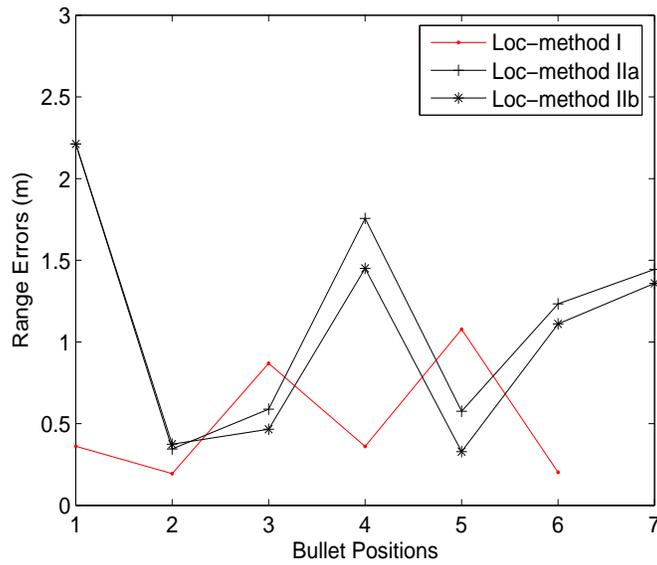


Figure 4. Locating errors of various localization methods.