Multi-Beam Scheduling for Unmanned Aerial Vehicle Networks

Batu K. Chalise, Yimin D. Zhang, and Moeness G. Amin

Wireless Communications and Positioning Laboratory
Center for Advanced Communications
Villanova University, Villanova, PA 19085, USA

Abstract—**In this paper, we consider the design of an optimum beamforming and scheduling scheme at a multi-antenna UAV central node (UAV-C), which collects information from several other single-antenna distributed UAV nodes (UAV-D). When the channels between UAV-C and UAV-D are both perfectly known, the optimum strategy is to employ minimum mean-square-error (MMSE) beamforming in conjuction with a computationally efficient combinatorial approach. For the case where the channels between UAV-C and UAV-D are subject to channel estimation errors, a stochastic robust design approach is proposed. In this case, it is shown that the optimum strategy is to use robust MMSE beamformers in a combinatorial but efficient manner. Simulation results show that robust beamforming plus scheduling provides significant gains in average throughput over non-robust beamformers with scheduling, and the robust and non-robust beamformers without scheduling. It is observed that, although multi-beam beamforming with scheduling outperforms beamforming without scheduling, the importance of the robust beamformer should not be overlooked as the beamformer and scheduler designed with the assumption of perfect channel knowledge result in significant reduction of throughput in cases where channel estimation is erroneous.**

Index Terms—**Multi-beam Scheduling and beamforming, imperfect channel, robust beamforming, UAV networks**

I. Introduction

Recently, unmanned aerial vehicles (UAVs) have attracted significant interests in civilian and military applications, including reconnaissance, border patrol, traffic control, and forest fire monitoring [1], [2], [3]. UAVs have also found useful to relay information between distant platforms so as to maintain reliable wireless communication links [4], [5], [6]. In particular, the UAV relays become most beneficial when the line-of-sight (LOS) between the source and destination nodes is obstructed by mountains or buildings, which are difficult to directly communicate to each other.

With the use of advanced wireless technology and micro-electromechanical systems, swarms of small UAVs are expected to be deployed for more challenging missions. As such, multiple source nodes may often compete the wireless channels, and efficient transmission schemes are desired to enhance the quality of service [7]. Upon appropriate resource management, the concurrent transmission of information from multiple source nodes in a wireless network have shown to achieve further improved spectral and power efficiency [8], [9], [10].

In this paper, we consider optimum beamforming and scheduling in a UAV network with an objective to maximize the system sum capacity or throughput. A multi-antenna UAV central node (UAV-C) receives information from N distributed UAVs (UAV-D). Based on either perfect or imperfect knowledge of channels between the UAV-C and UAV-D nodes, beamforming and scheduling tasks are executed at the UAV-C node. In particular, the UAV-C node tunes its K beams to receive signals from N UAV-D nodes, where $K \leq N$. When the channels are perfectly known, it is shown that the optimum strategy is to employ the minimum mean-square-error (MMSE) beamforming in conjunction with a combinatorial approach. For the case, where the channels are subject to estimation errors, a stochastic robust design approach [11], [12] is proposed to make the beamformer robust against channel estimation errors. In this case, it is shown that the optimum strategy is to use robust MMSE beamformers, also in a combinatorial approach. Because K is usually smaller than M, i.e., the number of antennas at the UAV-C node, the aforementioned combinatorial approach can be efficiently handled by brute-force search. Average sum capacity results obtained from computer simulations give several important insights. First, it is clearly seen that beamforming with scheduling provides significant performance gains over beamforming without scheduling. Second, if the channels are subject to estimation errors, beamforming with scheduling may not suffice. In order to make the beamforming with scheduling robust against channel estimation errors, the robust design of beamformer is preferred. In essence, robust beamforming with scheduling provides significant gains in average throughput over the non-robust beamformers with scheduling, and the robust and non-robust beamformers without scheduling.

The rest of the paper is organized as follows. In Section II, the system model is presented, whereas in Section III, an optimum beamforming and scheduling approach is proposed for the case where the channels are perfectly known. The beamforming and scheduling design of Section III is then extended in Section IV to a case where channels are imperfectly known. Simulation results are presented in Section V to illustrate the superior performance of the proposed beamforming and scheduling method. Conclusions are drawn in Section VI.

Notations: The following notations are used in this paper. A lower (upper) case bold letter denotes a vector (matrix). $(.)^H$ and $(.)^T$ respectively denote Hermitian transpose and transpose operations. $\mathrm{E}[\cdot]$ denotes the expectation operator.

II. System Model

As shown in Fig. 1, we consider a multi-beam node, denoted as D, which consists of M antennas. Node D receives signals from N source nodes, denoted as S_i ($i = 1, 2, \cdots, N$). The source nodes are single-antenna nodes.

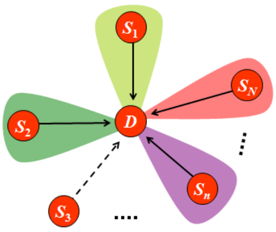

Fig. 1. Multi-beam adaptive beamformer for an UAV network (Solid lines indicate beams that are assigned to serve the nodes, whereas the dashed line indicates that the node cannot be served.)

Denote $u_i \in \{1, 0\}$ as the action indicator of source node S_i for $i = 1, ..., N$, and denote $\mathbf{u} = [u_1, ..., u_N]^T$. u_i takes a value of 1 when node S_i is in the "transmission" mode, whereas its value is 0 when node S_i in the "non-transmission" mode. The signal vector received at node D is expressed as

$$\mathbf{x} = \sum_{i=1}^{N} u_i h_i \sqrt{P_i} s_i \mathbf{a}_i + \mathbf{n} \tag{1}$$

where s_i and P_i are, respectively, the signal and power transmitted from node S_i, and h_i is the propagation channel gain between nodes S_i and D. We assume that $s_i, \forall i$, have a unit power and are mutually uncorrelated, i.e., $\mathrm{E}[s_i s_k^*] = 0$ for $i \neq k$, and $\mathrm{E}[|s_i|^2] = 1$. In addition, $\mathbf{a}_i$ is the $M \times 1$ steering vector corresponding to the source node S_i. h_i and $\mathbf{a}_i$ are considered unchanged during the coherent processing time. $\mathbf{n}$ is the $M \times 1$ additive noise vector whose elements are assumed to be independent and identically distributed (i.i.d.) complex Gaussian variables with zero-mean and variance σ^2.

III. Beamforming and Scheduling under Perfect Channel Knowledge

Assume that the node D adaptively forms K beams ($K \leq M$). Each beam is formed to receive signals transmitted from a different source node. We use a notation $i = b(k)$ to denote that the kth beamformer is tuned to receive signal from S_i. As such, the received signal from the kth beamformer is expressed as

$$\begin{aligned} r_{b(k)} &= \mathbf{w}_{b(k)}^H \mathbf{x} \\ &= u_i \sqrt{P_i} s_i \mathbf{w}_{b(k)}^H \mathbf{h}_i \\ &\quad + \sum_{l=1, l\neq i}^{N} u_l \sqrt{P_l} s_l \mathbf{w}_{b(k)}^H \mathbf{h}_l + \mathbf{w}_{b(k)}^H \mathbf{n} \end{aligned} \tag{2}$$

where we have defined $\mathbf{h}_i = h_i \mathbf{a}_i, \forall i = 1, \cdots, N$. The first term at the right-hand side corresponds to the desired signal from S_i, whereas the second term is the interference from other nodes, and the last term is the additive noise. The mean-square-error (MSE), $\mathrm{MSE}_i = \mathrm{E}\left\{|r_{b(k)} - \mathbf{w}_{b(k)}^H \mathbf{x}|^2\right\}$, can be expressed as

$$\begin{aligned} \mathrm{MSE}_i = 1 + \mathbf{w}_{b(k)}^H \left(u_i P_i \mathbf{h}_i \mathbf{h}_i^H + \sum_{l=1, l\neq i}^{N} u_l P_l \mathbf{h}_l \mathbf{h}_l^H + \sigma^2 \mathbf{I}_M \right) \\ \times \mathbf{w}_{b(k)} - u_i \sqrt{P_i} \left(\mathbf{w}_{b(k)}^H \mathbf{h}_i + \mathbf{h}_i^H \mathbf{w}_{b(k)} \right) \end{aligned} \tag{3}$$

We consider the sum capacity as a performance metric. Noting that $\mathrm{MMSE} = \frac{1}{1+\mathrm{SINR}}$ [13], where MMSE and SINR, are respectively, the MMSE and signal-to-interference plus noise ratio (SINR), the sum capacity (as a function of $\mathbf{u}$) can be expressed as

$$C(\mathbf{u}) = \sum_{i=1}^{N} \log_2(\mathrm{MMSE}_i^{-1}). \tag{4}$$

Because each beam is tuned to a specific user, MSE_i is decoupled in terms of $\mathbf{w}_{b(k)}, \forall k$, for a given $\mathbf{u}$. As a result, the optimum beamformers can be obtained by individually minimizing each MSE_i. For $\mathbf{u} = \mathbf{u}_0 = [u_{1,0}, \cdots, u_{N,0}]^T$, we obtain the following MMSE beamformer

$$\mathbf{w}_{b(k)} = \sqrt{P}_i u_{i,0} \mathbf{R}_\mathrm{T}^{-1} \mathbf{h}_i, \forall k, i, i = b(k) \tag{5}$$

where

$$\mathbf{R}_\mathrm{T} = \mathrm{E}\left\{\mathbf{x}\mathbf{x}^H\right\} = u_{i,0} P_i \mathbf{h}_i \mathbf{h}_i^H + \sum_{l=1, l\neq i}^{N} u_{l,0} P_l \mathbf{h}_l \mathbf{h}_l^H + \sigma^2 \mathbf{I}_M. \tag{6}$$

Substituting the MMSE beamformer (5) into (3), the achieved MMSE for the ith node is

$$\mathrm{MMSE}_i = 1 - u_{i,0} P_i \mathbf{h}_i^H \mathbf{R}_\mathrm{T}^{-1} \mathbf{h}_i = \frac{1}{1 + u_{i,0} P_i \mathbf{h}_i^H \mathbf{R}_{\mathrm{I},i}^{-1} \mathbf{h}_i} \tag{7}$$

where

$$\mathbf{R}_{\mathrm{I},i} = \mathbf{R}_\mathrm{T} - u_{i,0} P_i \mathbf{h}_i \mathbf{h}_i^H. \tag{8}$$

The sum capacity is then given by

$$\begin{aligned} C(\mathbf{u}_0) = \sum_{i=1}^{N} \log_2 \Bigg\{ 1 + u_{i,0} P_i \mathbf{h}_i^H \left[\sum_{l=1, l\neq i}^{N} u_{l,0} P_l \mathbf{h}_l \mathbf{h}_l^H + \sigma^2 \mathbf{I}_M \right]^{-1} \\ \mathbf{h}_i \Bigg\}. \end{aligned} \tag{9}$$

The objective of the multi-beam scheduling optimization problem is to maximize the sum capacity under the constraint that K beams can be formed. The corresponding optimization problem for scheduling can be expressed as

$$\begin{aligned} \mathcal{P}_1 : \quad & \max_{\{\mathbf{u} \in \{1,0\}^{N\times 1}\}} C(\mathbf{u}) \\ & \text{s.t. } \mathbf{u}^T \mathbf{1} \leq K \end{aligned} \tag{10}$$

where $\mathbf{1}$ is the vector whose elements are all 1. Since the elements of $\mathbf{u}$ are binary integers, it is clear that $\mathcal{P}_1$ is a integer programming problem, which is known to be NP hard. Several methods, such as Lagrangian, semidefinite programming and completely positive programming relaxations can be employed to solve $\mathcal{P}_1$ suboptimally [14]. Because $K \leq M$ and the major computational complexity associated with the optimum beamformer calculation is only inversion of a single $M \times M$

matrix $\mathbf{R}_{\mathrm{T}}$, the joint beamforming and scheduling can be efficiently solved by brute-force. This means that the sum capacity can be maximized by obtaining MMSE beamformers for 2^K combinations of $\mathbf{u}$ and selecting the combination that gives the maximum sum capacity.

IV. Robust Design of Beamforming and Scheduling

In practice, the channel estimates may not be perfect. This will yield degradation of the output MSE performance. The strategies of scheduling the source nodes should also be changed accordingly. In the presence of channel estimation errors, the actual $\mathbf{h}_i$ and its estimate $\hat{\mathbf{h}}_i$ can be related as

$$\mathbf{h}_i = \hat{\mathbf{h}}_i + \mathbf{e}_i, i = 1, \cdots, N \tag{11}$$

where $\mathbf{e}_i, \forall i$, are the estimation errors, whose exact values are unknown. If the beamformer is designed solely on the basis of $\hat{\mathbf{h}}_i$, the beamformer's performance is inevitable to degrade. However, if an upper bound on the Euclidean norm or the statistical distribution of $\mathbf{e}_i$ can be exploited, the performance of the beamformer can be made robust against unknown $\mathbf{e}_i$. The approach based on the deterministic upper bound of the error norm results in worst-case performance based robust design [15], [16], whereas the approach based on the statistics of error results in stochastic robust design [11], [12]. The objective of the worst-case approach is to solve the beamformer design problem for the worst-case errors. The resulting beamforming and scheduling design, however, turns to be a computationally demanding optimization problem. In this paper, we employ stochastic robust design for the considered UAV network, which has the same computational complexity as the beamformer design with perfect channel knowledge and requires only the knowledge of the second-order statistics of the channel estimation errors.

In the presence of the estimation errors, the received signal corresponding to the kth beamformer can be expressed as

$$\begin{aligned}\tilde{r}_{b(k)} &= \mathbf{w}_{b(k)}^H \Big(u_i \sqrt{P}_i (\hat{\mathbf{h}}_i + \mathbf{e}_i) s_i \\ &\quad + \sum_{l=1, l\neq i}^{N} u_l \sqrt{P}_l (\hat{\mathbf{h}}_l + \mathbf{e}_l) s_l + \mathbf{n} \Big).\end{aligned} \tag{12}$$

The corresponding MSE, $\mathrm{MSE}_i^{\mathrm{r}} = E\left\{|\tilde{r}_{b(k)} - s_i|^2\right\}$, is given by

$$\begin{aligned}\mathrm{MSE}_i^{\mathrm{r}} &= 1 + u_i P_i \mathbf{w}_{b(k)}^H (\hat{\mathbf{h}}_i + \mathbf{e}_i)(\hat{\mathbf{h}}_i + \mathbf{e}_i)^H \mathbf{w}_{b(k)} \\ &\quad + \sum_{l=1, l\neq i}^{K} u_l P_l \mathbf{w}_{b(k)}^H (\hat{\mathbf{h}}_l + \mathbf{e}_l)(\hat{\mathbf{h}}_l + \mathbf{e}_l)^H \mathbf{w}_{b(k)} \\ &\quad - u_i \sqrt{P}_i \left(\mathbf{w}_{b(k)}^H (\hat{\mathbf{h}}_i + \mathbf{e}_i) + (\hat{\mathbf{h}}_i + \mathbf{e}_i)^H \mathbf{w}_{b(k)} \right) \\ &\quad + \sigma^2 ||\mathbf{w}_{b(k)}||^2.\end{aligned} \tag{13}$$

In the stochastic robust design approach, the objective function is averaged over all possible realizations of the estimation errors. Towards this end, the average MSE, $\overline{\mathrm{MSE}}_i^{\mathrm{r}} = \mathrm{E}_{\{\mathbf{e}_i, \forall i\}}\left\{\mathrm{MSE}_i^{\mathrm{r}}\right\}$, is expressed as

$$\begin{aligned}\overline{\mathrm{MSE}}^{\mathrm{r}}{}_i &= 1 + u_i P_i \mathbf{w}_{b(k)}^H (\hat{\mathbf{h}}_i \hat{\mathbf{h}}_i^H + \mathbf{R}_{\mathbf{e}_i}) \mathbf{w}_{b(k)} \\ &\quad + \mathbf{w}_{b(k)}^H \left\{ \sum_{l=1, l\neq i}^{N} u_l P_l (\hat{\mathbf{h}}_l \hat{\mathbf{h}}_l^H + \mathbf{R}_{\mathbf{e}_l}) + \sigma^2 \mathbf{I}_M \right\} \mathbf{w}_{b(k)} \\ &\quad - u_i \sqrt{P}_i \left(\mathbf{w}_{b(k)}^H \hat{\mathbf{h}}_i + \hat{\mathbf{h}}_i^H \mathbf{w}_{b(k)} \right)\end{aligned} \tag{14}$$

where

$$\mathbf{R}_{\mathbf{e}_i} = \mathrm{E}\left\{ \mathbf{e}_i \mathbf{e}_i^H \right\}. \tag{15}$$

As in the case without estimation errors, $\overline{\mathrm{MSE}}^{\mathrm{r}}{}_i$ is decoupled in terms of $\mathbf{w}_{b(k)}, \forall k$, for a given $\mathbf{u}$. The beamformer that minimizes $\overline{\mathrm{MSE}}^{\mathrm{r}}{}_i$ is the MMSE beamformer, which can be expressed as

$$\mathbf{w}_{b(k)}^{\mathrm{r}} = \sqrt{P}_i u_{i,0} \tilde{\mathbf{R}}_{\mathrm{T}}^{-1} \hat{\mathbf{h}}_i, \forall i, k, i = b(k) \tag{16}$$

where the superscript 'r' in $\mathbf{w}_{b(k)}^{\mathrm{r}}$ denotes that the beamformer is robust, and

$$\begin{aligned}\tilde{\mathbf{R}}_{\mathrm{T}} &= u_{i,0} P_i \hat{\mathbf{h}}_i \hat{\mathbf{h}}_i^H + \sum_{l=1, l\neq i}^{N} u_{l,0} P_l \hat{\mathbf{h}}_l \hat{\mathbf{h}}_l^H + u_{i,0} P_i \mathbf{R}_{\mathbf{e}_i} \\ &\quad + \sum_{l=1, l\neq i}^{N} u_{l,0} P_l \mathbf{R}_{\mathbf{e}_l} + \sigma^2 \mathbf{I}_M.\end{aligned} \tag{17}$$

When the estimation error is mutually uncorrelated, $\mathbf{R}_{\mathbf{e}_i}$ is diagonal. As such, the inclusion of $\mathbf{R}_{\mathbf{e}_i}$ amounts to diagonal loading. The MMSE achieved with the beamformer (16) is expressed as

$$\overline{\mathrm{MMSE}}^{\mathrm{r}}{}_i = 1 - u_{i,0} P_i \hat{\mathbf{h}}_i^H \tilde{\mathbf{R}}_{\mathrm{T}}^{-1} \hat{\mathbf{h}}_i = \frac{1}{1 + u_{i,0} P_i \hat{\mathbf{h}}_i^H \tilde{\mathbf{R}}_{\mathrm{I},i}^{-1} \hat{\mathbf{h}}_i} \tag{18}$$

where

$$\tilde{\mathbf{R}}_{\mathrm{I},i} = \tilde{\mathbf{R}}_{\mathrm{T}} - u_{i,0} P_i \hat{\mathbf{h}}_i \hat{\mathbf{h}}_i^H \tag{19}$$

is the interference-plus-noise covariance matrix. The sum capacity is then defined as

$$C^{\mathrm{r}}(\mathbf{u}) = \sum_{i=1}^{N} \log_2 \left(\left(\overline{\mathrm{MMSE}}^{\mathrm{r}}{}_i \right)^{-1} \right). \tag{20}$$

Substituting (18) into (20), the sum capacity for the robust beamformer is given by

$$\begin{aligned}C^{\mathrm{r}}(\mathbf{u}_0) &= \sum_{i=1}^{N} \log_2 \Bigg\{ 1 + u_{i,0} P_i \hat{\mathbf{h}}_i^H \Bigg[\sum_{l=1, l\neq i}^{N} u_{l,0} P_l \Big(\hat{\mathbf{h}}_l \hat{\mathbf{h}}_l^H \\ &\quad + \mathbf{R}_{\mathbf{e}_l} \Big) + u_{i,0} P_i \mathbf{R}_{\mathbf{e}_i} + \sigma^2 \mathbf{I}_M \Bigg]^{-1} \hat{\mathbf{h}}_i \Bigg\}.\end{aligned} \tag{21}$$

Consequently, the optimization problem for scheduling can be expressed as

$$\begin{aligned}\mathcal{P}_2 : \quad & \max_{\left\{\mathbf{u} \in \{1,0\}^{N\times 1}\right\}} C^{\mathrm{r}}(\mathbf{u}) \\ & \text{s.t. } \mathbf{u}^T \mathbf{1} \leq K.\end{aligned} \tag{22}$$

The optimization problem $\mathcal{P}_2$ is the binary integer programming problem as $\mathcal{P}_1$ and cannot be efficiently solved. However, as the computational cost involved with the robust beamformer design is same as in the case without estimation errors and $K \leq M$, the joint beamforming and scheduling can be still efficiently solved by the brute-force approach. This means that the sum capacity can be maximized by obtaining robust MMSE beamformers for 2^K combinations of $\mathbf{u}$ and selecting the combination that gives the maximum sum capacity. The following remark is now in order.

Remark 1: When the beamformer (5), obtained under the assumption that the channels are perfectly known, is used for the case with channel estimation errors, i.e., in (14), we obtain the MMSE value for the *naive design*. This can be expressed as

$$\mathrm{MMSE}_i^{\mathrm{Nd}} = 1 - u_{i,0} P_i \hat{\mathbf{h}}_i^H \mathbf{R}_{\mathrm{T}}^{-1} \left(\mathbf{I}_M - \mathbf{E}\mathbf{R}_{\mathrm{T}}^{-1}\right) \hat{\mathbf{h}}_i \tag{23}$$

where

$$\mathbf{E} = \sum_{l=1}^{N} u_{l,0} P_l \mathbf{R}_{\mathbf{e}_l}. \tag{24}$$

Proposition 1: The relationship between $\overline{\mathrm{MMSE}}_i^{\mathrm{r}}$ and $\mathrm{MMSE}_i^{\mathrm{Nd}}$ is given by $\overline{\mathrm{MMSE}}^{\mathrm{r}}_i \leq \mathrm{MMSE}_i^{\mathrm{Nd}}$ for all i, k, where $i = b(k)$.

Proof: Assume that $\overline{\mathrm{MMSE}}_i^{\mathrm{r}} > \mathrm{MMSE}_i^{\mathrm{Nd}}$ is true. Then, we have

$$\begin{aligned} &\hat{\mathbf{h}}_i^H \tilde{\mathbf{R}}_{\mathrm{T}}^{-1} \hat{\mathbf{h}}_i < \hat{\mathbf{h}}_i^H \mathbf{R}_{\mathrm{T}}^{-1} \left(\mathbf{I}_M - \mathbf{E}\mathbf{R}_{\mathrm{T}}^{-1}\right) \hat{\mathbf{h}}_i \\ &\Rightarrow \hat{\mathbf{h}}_i^H (\mathbf{R}_{\mathrm{T}} + \mathbf{E})^{-1} \hat{\mathbf{h}}_i < \hat{\mathbf{h}}_i^H \mathbf{R}_{\mathrm{T}}^{-1} \left(\mathbf{I}_M - \mathbf{E}\mathbf{R}_{\mathrm{T}}^{-1}\right) \hat{\mathbf{h}}_i. \end{aligned} \tag{25}$$

Using the matrix inversion lemma, $(\mathbf{R}_{\mathrm{T}} + \mathbf{E})^{-1}$ can be expressed as

$$(\mathbf{R}_{\mathrm{T}} + \mathbf{E})^{-1} = \mathbf{R}_{\mathrm{T}}^{-1} - \mathbf{R}_{\mathrm{T}}^{-1}(\mathbf{I}_M + \mathbf{E}\mathbf{R}_{\mathrm{T}}^{-1})^{-1}\mathbf{E}\mathbf{R}_{\mathrm{T}}^{-1}. \tag{26}$$

Substituting (26) into (25) and with some simplifications, (25) can be expressed as

$$\hat{\mathbf{h}}_i^H \mathbf{R}_{\mathrm{T}}^{-1} \mathbf{E} \mathbf{R}_{\mathrm{T}}^{-1} \hat{\mathbf{h}}_i < \hat{\mathbf{h}}_i^H \mathbf{R}_{\mathrm{T}}^{-1} (\mathbf{I}_M + \mathbf{E}\mathbf{R}_{\mathrm{T}}^{-1})^{-1} \mathbf{E}\mathbf{R}_{\mathrm{T}}^{-1} \hat{\mathbf{h}}_i. \tag{27}$$

Note that $\mathbf{E}$ is a positive semidefinite hermitian matrix, whereas $\mathbf{R}_{\mathrm{T}}$ is a positive definite matrix. Moreover, we have $\mathrm{eig}(\mathbf{E}\mathbf{R}_{\mathrm{T}}^{-1}) = \mathrm{eig}(\mathbf{R}_{\mathrm{T}}^{-1}\mathbf{E})$, where $\mathrm{eig}(\cdot)$ stands for the eigenvalues of a matrix. Then , according to Theorem 7.6.3 of [17], $\mathbf{E}\mathbf{R}_{\mathrm{T}}^{-1}$ is positive semidefinite. Define the eigendecomposition (ED) of $\mathbf{E}\mathbf{R}_{\mathrm{T}}^{-1}$ as $\mathbf{E}\mathbf{R}_{\mathrm{T}}^{-1} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^H$, where $\mathbf{U}$ is a unitary matrix and $\mathbf{\Lambda}$ is a diagonal matrix with the non-negative eigenvalues of $\mathbf{E}\mathbf{R}_{\mathrm{T}}^{-1}$. Substituting this ED into (27) and after some straightforward steps, (27) can be expressed as

$$\hat{\mathbf{h}}_i^H \mathbf{R}_{\mathrm{T}}^{-1} \mathbf{U} [\mathbf{I}_M - (\mathbf{I} + \mathbf{\Lambda})^{-1}] \mathbf{\Lambda} \mathbf{U}^H \hat{\mathbf{h}}_i < 0. \tag{28}$$

It is clear that the matrix $\mathbf{U}[\mathbf{I}_M - (\mathbf{I}+\mathbf{\Lambda})^{-1}]\mathbf{\Lambda}\mathbf{U}^H$ is a Hermitian positive semidefinite matrix. Consequently, $\mathbf{R}_{\mathrm{T}}^{-1}\mathbf{U}[\mathbf{I}_M - (\mathbf{I} + \mathbf{\Lambda})^{-1}]\mathbf{\Lambda}\mathbf{U}^H$ is a positive semidefinite [Theorem 7.6.3 of [17]] matrix. This contradicts with the fact that (28) is true, which in turn contradicts with the assumption that $\overline{\mathrm{MMSE}}_i^{\mathrm{r}} > \mathrm{MMSE}_i^{\mathrm{Nd}}$ is true. This completes the proof of Proposition 1. □.

Because $\overline{\mathrm{MMSE}}_i^{\mathrm{r}} \leq \mathrm{MMSE}_i^{\mathrm{Nd}}, \forall i, k, i = b(k)$, it is clear that $C^{\mathrm{r}}(\mathbf{u}_0)$ is larger than $C^{\mathrm{Nd}}(\mathbf{u}_0)$, where $C^{\mathrm{Nd}}(\mathbf{u}_0) = \sum_{i=1}^{N} \log_2 \left(\left(\mathrm{MMSE}_i^{\mathrm{Nd}} \right)^{-1} \right)$.

V. Numerical Results

In this section, we compare the performance of the proposed scheduling with that of the system that does not run the scheduling algorithm (i.e., all source nodes are in the transmission mode), both for the cases with perfect and imperfect channel knowledge. The performance of the proposed robust beamformer is compared with the *naive* design which considers the estimated channels as the true channels, both for the cases with and without scheduling. For all simulation results, we take $K = M$, a uniform circular array (UCA) of M elements with a radius of circle as a half-wavelength, and $P_i = 1, \forall i$. All results are obtained by averaging over 1000 random channel realizations which are drawn from zero-mean complex Gaussian distribution with a variance σ_{h}^2.

A. No Channel Estimation Errors

We first discuss the results obtained for the case with the perfect channel knowledge. The average sum capacity as a

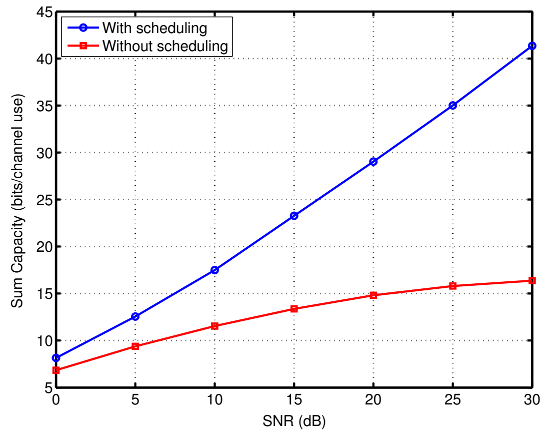

Fig. 2. Sum capacity versus SNR

function of signal-to-noise ration (SNR) is shown in Fig. 2 for cases without and with scheduling. The SNR is defined as $\log_{10}\left(\frac{1}{\sigma^2}\right)$, where σ^2 is varied. We take $M = 4$, $N = 6$ and $\sigma_{\mathrm{h}}^2 = 1$ for this figure. It can be seen from Fig. 2 that scheduling provides significant performance improvements over the case without scheduling. Because the scheme without scheduling observes significant interference, it is seen that the sum capacity gets saturated very fast as the SNR increases. However, for the case with scheduling, the sum capacity improves significantly when SNR increases, suggesting that the scheduler suppresses interference very effectively. In Fig. 3, the sum capacity versus M is displayed for the cases with and without the proposed scheduling. We take SNR of 20 dB and $\sigma_{\mathrm{h}}^2 = 1$. It can be observed from this figure that the most significant gains of the proposed scheduling occur when $2 \leq M \leq 6$. However, when M goes above 6, the performance gap between the schemes with and without scheduling decreases. This could be understood from the fact that as M approaches N and becomes larger than N, the interference suppression capability of the antenna array improves so that the gains obtained from scheduling becomes minimal. In other words, this result shows that it is important to combine beamforming with scheduling when $M << N$.

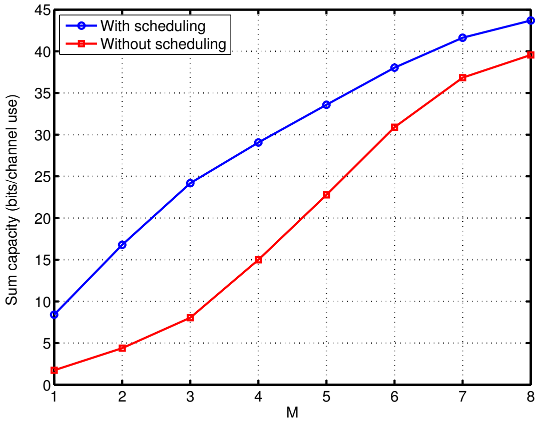

Fig. 3. Sum capacity versus M (perfect channel knowledge)

B. With Channel Estimation Errors

We now discuss the results obtained for the case when channels are subject to estimation errors. In Fig. 4, the sum capacity of four different schemes are compared, namely, the robust and naive beamformer designs with and without scheduling. The variance of channel estimation errors is taken as $\sigma_e^2 = 0.02$, whereas σ_{h}^2 and N are set to 1 and 6, respectively. When M increases, the performance of all methods improves. The robust design method with scheduling outperforms all other designs for all M. The performance gap between the robust design with and without scheduling decreases after $M \geq N$. Moreover, although the naive design with scheduling outperforms both robust and naive designs without scheduling for $M < 5$, it is observed that the performance of the former method becomes inferior to the robust method without scheduling when M goes beyond 5.

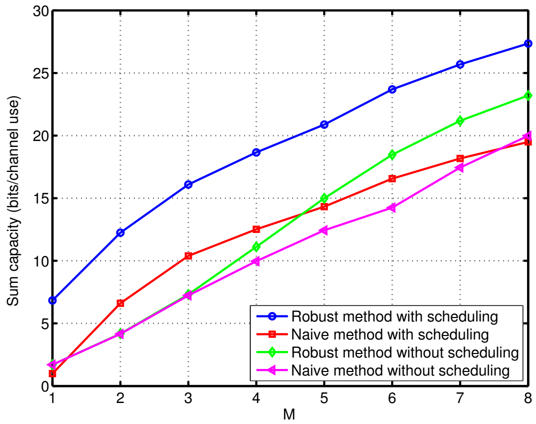

Fig. 4. Sum capacity versus M in the presence of channel estimation errors

As in Fig. 4, the sum capacity of different methods are compared in Fig. 5, but for different values of N by taking $M = 4$. We take $\sigma_e^2 = 0.03$ and $\sigma_{\mathrm{h}}^2 = 1$. It can be observed from this figure that, when N increases, the sum capacity decreases after $N \geq 3$, for both robust and naive beamformers without scheduling. However, when scheduling is employed, sum capacity of both methods increases for all considered values of N. This means that scheduling can exploit multi-user diversity by properly handling the interference scenario.

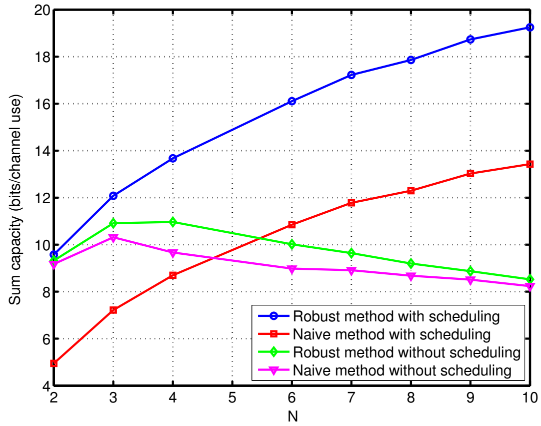

Fig. 5. Sum capacity versus N ($M = 4$) in the presence of channel estimation errors

VI. Conclusion

We have proposed an optimum beamforming and scheduling method for a UAV network, where a multi-antenna UAV central node receives information from multiple distributed UAV nodes. Considering that the UAV central node can estimate all channels with and without estimation errors, the optimum beamforming and scheduling problem is solved by obtaining the MMSE beamformers in an efficient approach. Simulation results show that, although beamforming with scheduling outperforms beamforming without scheduling, robust design of beamformer is required to avoid degradation of the beamforming with scheduling method.

References

[1] D. Casbeer, S. Li, R. Beard, T. McLain, and R. Mehra, "Forest fire monitoring with multiple small UAVs," in *Proc. American Control Conference*, (Portland, OR), pp. 3530–3535, June 2005.

[2] H. Chao, Y. Cao, and Y. Chen, "Autopilots for small fixed wing unmanned air vehicles: a survey," in *Proc. Mechatronics and Automation*, (Harbin, China), pp. 3144–3149, Aug. 2007.

[3] H. Chao, M. Baumann, A. Jensen, Y. Chen, Y. Cao, W. Ren, and M. McKee, "Band-reconfigurable multi-UAV-based cooperative remote sensing for real-time water management and distributed irrigation control," in *Proc. IFAC World Congress*, (Seoul, Korea), July 2008.

[4] R. Palat, A. Annamalai, and J. Reed, "Cooperative relaying for ad-hoc ground networks using swarm UAVs," in *Proc. MilCom*, (Atlantic City, NJ), pp. 1588–1594, Oct. 2005.

[5] I. Abualhaol and M. Matalgah, "Outage probability analysis in a cooperative UAVs network over Nakagami-m fading channels," in *Proc. Vehicular Technology Conference*, (Montreal, Quebec), pp. 1–4, Sep. 2006.

[6] X. Li and Y. D. Zhang, "Multi-source cooperative communications using multiple small relay UAVs," in *Proc. Globecom Workshop on Wireless Networking for Unmanned Aerial Vehicles Program*, (Miami, FL), Dec. 2010.

[7] Y. Cai, F. R. Yu, J. Li, Y. Zhou, and L. Lamont, "Medium access control for unmanned aerial vehicle (UAV) ad-hoc networks with full-duplex radios and multi-packet reception capability," *IEEE Trans. Veh. Technol.*, vol. 62, pp. 390–394, Jan. 2013.

[8] Y. Zhang, X. Li, and M. G. Amin, *Mobile ad hoc networks exploiting multi-beam antennas*. in C. Sun, J. Cheng, and T. Ohira (eds.), Handbook on Advancements in Smart Antenna Technologies for Wireless Networks, Hersey, PA: Information Science Reference, 2008.

[9] X. Li, Y. Zhang, and M. G. Amin, “Priority-based access schemes and throughput performance in wireless networks exploiting multibeam antennas,” *IEEE Trans. Veh. Technol.*, vol. 58, pp. 3569–3578, Sept. 2009.

[10] X. Li, Y. Zhang, and M. G. Amin, “Node throughput analysis of decentralized wireless networks using multibeam antennas in multipath environments,” *Wireless Personal Commun.*, vol. 50, no. 4, pp. 447–456, 2009.

[11] X. Zhang, D. P. Palomar, and B. Ottersten, “Statistically robust design of linear MIMO transceivers,” *IEEE Trans. Signal Proc.*, vol. 56, pp. 3678–3689, Aug. 2008.

[12] B. K. Chalise and L. Vandendorpe, “Joint linear processing for an amplify-and-forward MIMO relay channel with imperfect channel state information,” *EURASIP J. Adv. Signal Proc.*, Article ID 640186 2010.

[13] D. Guo, S. Shamai, and S. Verdu, “Mutual information and minimum mean-square error in Gaussian channels,” *IEEE Trans. Inform. Theory*, vol. 51, pp. 1261–1282, April 2005.

[14] L. Lovász and A. Schrijver, “Cones of matrices and set-functions, and 0-1 optimization,” *SIAM J. Optim.*, vol. 1, pp. 166–190, 1991.

[15] S. A. Vorobyov, A. B. Gershman, and Z.-Q. Luo, “Robust adaptive beamforming using worst-case performance optimization: A solution to the signal mismatch problem,” *IEEE Trans. Signal Proc.*, vol. 51, pp. 2257–2269, Sept. 2003.

[16] B. K. Chalise and L. Vandendorpe, “MIMO relay design for multipoint-to-multipoint communications with imperfect channel state information,” *IEEE Trans. Signal Proc.*, vol. 57, pp. 2785–2796, July 2009.

[17] R. A. Horn and C. R. Johnson, *Matrix Analysis*. Cambridge University Press, 1990.