Compressive sensing-based coprime array direction-of-arrival estimation

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Abstract: A coprime array has a larger array aperture as well as increased degrees-of-freedom (DOFs), compared with a uniform linear array with the same number of physical sensors. Therefore, in a practical wireless communication system, it is capable to provide desirable performance with a low-computational complexity. In this study, the authors focus on the problem of efficient direction-of-arrival (DOA) estimation, where a coprime array is incorporated with the idea of compressive sensing. Specifically, the authors first generate a random compressive sensing kernel to compress the received signals of coprime array to lower-dimensional measurements, which can be viewed as a sketch of the original received signals. The compressed measurements are subsequently utilised to perform high-resolution DOA estimation, where the large array aperture of the coprime array is maintained. Moreover, the authors also utilise the derived equivalent virtual array signal of the compressed measurements for DOA estimation, where the superiority of coprime array in achieving a higher number of DOFs can be retained. Theoretical analyses and simulation results verify the effectiveness of the proposed methods in terms of computational complexity, resolution, and the number of DOFs.

1 Introduction

Direction-of-arrival (DOA) estimation plays an important role in numerous kinds of wireless communication systems, such as ultra-dense cellular networks, mobile relay systems and secure communications [1–5]. Under the background of explosive demands for higher data rates and better reliability, accurate DOA estimation is desirable since it can provide fundamental information to guarantee the performance in terms of transmission speed, quality of service, bit error rate and so on. The uniform linear array (ULA) is one of the most commonly used array geometries in practical wireless communication systems. To obtain a higher resolution, more physical sensors are required to deploy since the resolution is directly determined by the array aperture. Meanwhile, in an ultra-dense cellular network, it is very common that the base station is required to cope with multiple users simultaneously. However, the achievable number of degrees-of-freedom (DOFs) of a ULA is limited by the number of physical sensors. Expanding the array aperture of a ULA is at the cost of increased hardware complexity and computational complexity. Therefore, meeting the increasing resolution and DOF demands in practical wireless communication systems while maintaining a moderate system complexity becomes a challenging but promising task.

Compared with the ULA, the sparse array provides a potential solution to reduce the system complexity. Several typical sparse array configurations have been proposed, such as minimum redundancy array [6], minimum hole array [7], and nested array [8]. Recently, a coprime array was proposed for systematical design of sparse arrays. Compared with ULA, a coprime array can provide a larger array aperture without the need of increasing the number of physical sensors. More importantly, it has been proved that the coprime array achieves $\mathcal{O}(MN)$ DOFs by using only $M + N – 1$ physical sensors [9]. Considering these advantages, the coprime array configuration is expected to be a good candidate for deploying future wireless communication systems and has attracted tremendous attentions in the past [10–15]. The exploitation of the extended array aperture of a coprime array can be classified into two different approaches. Methods in the first approach [11–13] directly utilise the physical array aperture where the coprime properties of the constituting sub-arrays are used to resolve ambiguity due to the sparse sub-array configurations. In the second approach [14–20], a virtual array is derived from the sparse physical array based on the difference coarray concept. By processing the derived equivalent virtual array signal, the DOF of DOA estimation is significantly increased since more nominal sensors in virtual domain can be utilised. Meanwhile, the basis mismatch problem has motivated the recent research interests on off-grid DOA estimation by using total variation norm [21], nuclear norm [22], and joint sparsity reconstruction [23], where the resolution performance of coprime array DOA estimation can be further improved. However, DOA estimation on virtual ULA with a large number of nominal sensors is of high-computational complexity.

Compressive sensing is one of the most exciting signal processing techniques in the past decade, which enables accurate signal recovery at sub-Nyquist sampling rates [24]. That is to say, under the framework of compressive sensing, the required hardware complexity and computational complexity can be effectively decreased. Hence, compressive sensing has been found useful in DOA estimation and many other applications [25–29]. In this paper, we introduce compressive sensing to the coprime array for efficient DOA estimation. Specifically, we first compress the signals received by the coprime array to lower-dimensional measurements through a random projection. The compressed measurements can be viewed as a sketch of the original coprime array received signals, whose information is not lost and the advantage of coprime array is maintained. To achieve high-resolution DOA estimation, the compressed measurements are employed to obtain the Capon spatial spectrum, where the large array aperture is maintained. Furthermore, the equivalent virtual array signal of the compressed measurements is derived for increased DOFs, where the DOA estimation is carried out under the framework of sparse signal reconstruction. As a beneficial result, the proposed DOA estimation methods using compressed measurements maintain the advantages offered by the coprime array, while the computational complexity is significantly reduced. Simulation results demonstrate the effectiveness of the proposed DOA estimation methods from the perspective of reduced complexity, improved resolution, and increased DOFs.
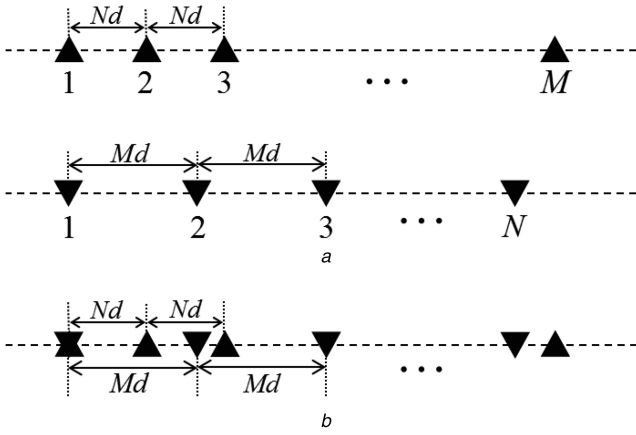
Fig. 1 *Coprime array geometry*
(a) Pair of sparsely-spaced ULAs, *(b)* Configuration of coprime array

The main contribution of this paper is threefold: (i) We compress the signal received at a coprime array to a lower-dimensional sketch through a random projection, and incorporate the compressed measurements to perform efficient DOA estimation; (ii) We devise a high-resolution DOA estimation method by using the compressed measurements, where the large array aperture of coprime array is maintained. The proposed method using coprime array outperforms the conventional methods using ULA in terms of estimation resolution. More importantly, it also works well when the number of snapshots is less than the number of physical sensors; (iii) We derive the compressed measurements to a virtual domain for sparse signal reconstruction, where an efficient DOA estimation method is designed from the perspective of increased DOFs.

The remainder of this paper is organised as follows. In Section 2, we describe the signal model of coprime array. In Section 3, we elaborate the design of proposed DOA estimation methods. Simulation results are presented and compared in Section 4. Finally, we conclude this paper in Section 5.

Notations: we use lower-case and upper-case boldface characters to represent the vectors and matrices throughout this paper. The superscripts $(\cdot)^{\mathrm{T}}$ and $(\cdot)^{\mathrm{H}}$, respectively, denote the transpose and Hermitian transpose operators, and $(\cdot)^*$ denotes the conjugate operator. The notation $E\{\cdot\}$ stands for the statistical expectation operator, vec($\cdot$) denotes the vectorisation process that stacks each column vector of a matrix one by one, and $\otimes$ denotes the Kronecker product. $\|\cdot\|_0$, $\|\cdot\|_1$, and $\|\cdot\|_2$ denote the ℓ_0-norm, ℓ_1-norm, and ℓ_2-norm, respectively. Finally, $\mathbf{0}$ denotes the zero vector, and $\boldsymbol{I}$ denotes the identity matrix with proper dimension.

2 Signal model of coprime array

We consider a pair of sparsely spaced ULAs as depicted in Fig. 1*a*. Denote M and N to be a pair of mutually coprime integers. The sparse ULA consisting of M elements has the sensors located at $\{0, N\lambda/2, 2N\lambda/2, \ldots, (M-1)N\lambda/2\}$, and the other sparse ULA consisting of N elements has the sensors located at $\{0, M\lambda/2, 2M\lambda/2, \ldots, (N-1)M\lambda/2\}$. Here, λ is the signal wavelength. The configuration of coprime array, shown in Fig. 1*b*, can be obtained by combining the pair of sparse ULAs with the first sensor of each sparse ULA overlapped. According to the property of coprime integers, the sensors of each sparse ULA do not overlap at other sensor positions. Therefore, the coprime array consists a total of $M+N$–1 distinct physical sensors.

Assuming D far-field uncorrelated narrowband signals from distinct directions $\{\theta_1, \theta_2, \ldots, \theta_D\}$, the received signal vector of the coprime array at time k can be modelled as

$$\boldsymbol{x}(k) = \sum_{d=1}^{D} \boldsymbol{a}(\theta_d)s_d(k) + \boldsymbol{n}(k) = \boldsymbol{As}(k) + \boldsymbol{n}(k), \qquad (1)$$

where $\boldsymbol{A} = [\boldsymbol{a}(\theta_1), \boldsymbol{a}(\theta_2), \ldots, \boldsymbol{a}(\theta_D)] \in \mathbb{C}^{(M+N-1)\times D}$ is the array steering matrix, $\boldsymbol{s}(k) = [s_1(k), s_2(k), \ldots, s_D(k)]^{\mathrm{T}}$ is the signal waveform vector, and $\boldsymbol{n}(k) \sim \mathscr{CN}(\mathbf{0}, \sigma_n^2 \boldsymbol{I})$ is a complex-valued Gaussian white noise vector independent of signals. Here, σ_n^2 is the noise power. The *d*th column of array steering matrix $\boldsymbol{A}$, i.e.

$$\boldsymbol{a}(\theta_d) = \left[1, \mathrm{e}^{-\mathrm{j}(2\pi/\lambda)u_2\sin(\theta_d)}, \ldots, \mathrm{e}^{-\mathrm{j}(2\pi/\lambda)u_{M+N-1}\sin(\theta_d)}\right]^{\mathrm{T}}, \qquad (2)$$

is the coprime array steering vector corresponding to θ_d, where u_i is the spacing between the first and the *i*th sensor of the coprime array.

3 DOA estimation based on compressive sensing coprime array signal

It can be seen from Fig. 1*b* that the coprime array consisting of $M+N$–1 physical sensors has an array aperture of $\max((M-1)N\lambda/2, (N-1)M\lambda/2)$. Since the resolution of DOA estimation is directly determined by the array aperture, we adopt a coprime array rather than a ULA for receiving the signals, and the received signal vector $\boldsymbol{x}(k)$ in (1) is referred to as the coprime array signal vector. Due to its sparse nature, the coprime array collects the signal information in a wider range of space, and thus provides a higher resolution for DOA estimation than a fully populated ULA consisting of the same number of physical sensors. In addition, the coprime array signal can also be incorporated to perform DOA estimation with increased DOFs, where an equivalent virtual array signal can be derived by calculating the difference coarray of the coprime array [9].

3.1 Compressive sensing coprime array signal

As we know, the computational complexity is one of the main concerns for algorithm implementation in practical wireless communication systems. To further reduce the computational complexity and develop an efficient DOA estimation method, we incorporate compressive sensing to the coprime array signal for DOA estimation while making maximum use of the superior advantages provided by the coprime array. In particular, with the idea of compressive sensing, a random compressive sensing kernel $\boldsymbol{\Phi} \in \mathbb{C}^{Q\times(M+N-1)}$ is applied to compress the coprime array signal vector $\boldsymbol{x}(k)$ as

$$\boldsymbol{y}(k) = \boldsymbol{\Phi x}(k) = \boldsymbol{\Phi}(\boldsymbol{As}(k) + \boldsymbol{n}(k)), \qquad (3)$$

where $Q \ll M+N-1$ determines the dimension of the compressed measurement $\boldsymbol{y}(k)$, and the elements of $\boldsymbol{\Phi}$ can be generated from a random distribution, such as Gaussian or Bernoulli distributions, if there is no available prior knowledge of desired signal [30]. By using such a random projection, the $(M+N$–1)-dimensional coprime array signal vector $\boldsymbol{x}(k)$ is compressed to a Q-dimensional compressed measurement vector $\boldsymbol{y}(k)$, which can be regarded as a *sketch* of vector $\boldsymbol{x}(k)$. The compression process enables us to perform DOA estimation from a lower-dimensional compressed measurement $\boldsymbol{y}(k)$ in an efficient manner since the essential information contained in $\boldsymbol{x}(k)$ is preserved.

The covariance matrix of compressed measurement $\boldsymbol{y}(k)$ can be computed as

$$\boldsymbol{R}_{yy} = E\left\{\boldsymbol{y}(k)\boldsymbol{y}^{\mathrm{H}}(k)\right\} = \boldsymbol{\Phi R}_{xx}\boldsymbol{\Phi}^{\mathrm{H}}, \qquad (4)$$

where

$$\boldsymbol{R}_{xx} = E\left\{\boldsymbol{x}(k)\boldsymbol{x}^{\mathrm{H}}(k)\right\} = \boldsymbol{A\Lambda A}^{\mathrm{H}} + \sigma_n^2\boldsymbol{I}$$
$$= \sum_{d=1}^{D} \sigma_d^2 \boldsymbol{a}(\theta_d)\boldsymbol{a}^{\mathrm{H}}(\theta_d) + \sigma_n^2\boldsymbol{I} \qquad (5)$$

is the covariance matrix of the coprime array signal vector $\boldsymbol{x}(k)$. Here, σ_d^2 is the power of *d*th source signal, and

$\mathbf{\Lambda} = E\{s(k)s^{\mathrm{H}}(k)\} = \mathrm{diag}[\sigma_1^2, ..., \sigma_D^2]$ is the diagonal matrix consisting of each source signal power. It should be emphasised that the additive Gaussian noise vector $n(k)$ is included in the coprime array signal vector $x(k)$ prior to the compression process. Therefore, the compressive sensing kernel $\mathbf{\Phi}$ operates on the signal component and the noise component simultaneously. If a non-orthogonal compressive sensing kernel is applied, the noise component $\sigma_n^2\mathbf{\Phi\Phi}^{\mathrm{H}}$ in R_{yy} will be distorted (non-diagonal). To avoid this problem, the compressive sensing kernel is chosen to be a row orthogonal matrix, namely, $\mathbf{\Phi\Phi}^{\mathrm{H}} = I$. Since the exact R_{yy} is unavailable in practice, it can be approximated by its sample version

$$\hat{R}_{yy} = \frac{1}{K}\sum_{k=1}^{K} y(k)y^{\mathrm{H}}(k) = \mathbf{\Phi}\hat{R}_{xx}\mathbf{\Phi}^{\mathrm{H}}, \tag{6}$$

where K is the number of snapshots, and

$$\hat{R}_{xx} = \frac{1}{K}\sum_{k=1}^{K} x(k)x^{\mathrm{H}}(k) \tag{7}$$

is the sample covariance matrix computed from the coprime array signal vectors $\{x(k), k = 1, 2, ..., K\}$. Based on the compressed measurements $\{y(k), k = 1, 2, ..., K\}$ along with their statistical parameters, we propose two efficient DOA estimation methods from the perspective of higher resolution and increased DOFs, respectively. Since $y(k)$ is a sketch of the coprime array signal vector, the superior advantages of coprime array can be retained.

3.2 DOA estimation with high resolution

We first take advantage of the large array aperture offered by the coprime array, and design a DOA estimation method with high-resolution by using COmpressed Measurements (HR-COM). The proposed HR-COM method can be implemented by computing the Capon spatial spectrum of the compressed measurements as

$$p(\theta) = \frac{1}{d^{\mathrm{H}}(\theta)\hat{R}_{yy}^{-1}d(\theta)}, \quad \theta \in [-90°, 90°], \tag{8}$$

where θ is the hypothetical direction on the predefined grid, and $d(\theta) = \mathbf{\Phi}a(\theta) \in \mathbb{C}^Q$ is the corresponding compressed steering vector. The DOAs can be estimated by searching for the peaks in the spatial spectrum $p(\theta)$.

Benefiting from compressive sensing, the computational complexity of the proposed HR-COM method using compressed coprime array signals for Capon spatial spectrum calculation is $\mathcal{O}(SQ^2)$, where $S \gg M + N - 1$ is the number of hypothetical directions in the spatial spectrum. In contrast, the computational complexity directly using coprime array signals for Capon spatial spectrum calculation is $\mathcal{O}(S(M + N)^2)$. Furthermore, if a fully populated ULA is employed for achieving the same array aperture of $\max(M(N-1)\lambda/2, N(M-1)\lambda/2)$ as the coprime array, its computational complexity increases to $\mathcal{O}(\max((MN)^3, S(MN)^2))$.

3.3 DOA estimation with increased DOFs

Increased number of DOFs is another important advantage of a coprime array, which can be realised by obtaining the equivalent virtual array signal in the derived virtual domain. Toward this end, we propose a DOA estimation method with increased DOFs by utilising the COmpressed measurements (ID-COM). The main idea of the proposed ID-COM method is to derive the correlation statistic of compressed measurements $\{y(k), k = 1, 2, ..., K\}$ to the virtual domain for sparse signal reconstruction. Specifically, the sample covariance matrix of compressed measurements $\hat{R}_{yy}$ can be vectorised as

$$\begin{aligned} z &= \mathrm{vec}(\hat{R}_{yy}) \\ &= (\mathbf{\Phi}^* \otimes \mathbf{\Phi})\mathrm{vec}(\hat{R}_{xx}) \\ &= (\mathbf{\Phi}^* \otimes \mathbf{\Phi})(\tilde{A}r + \sigma_n^2 i), \end{aligned} \tag{9}$$

where $\tilde{A} = [\tilde{a}(\theta_1), \tilde{a}(\theta_1), ..., \tilde{a}(\theta_D)] \in \mathbb{C}^{(M+N-1)^2 \times D}$ with $\tilde{a}(\theta_d) = a^*(\theta_d) \otimes a(\theta_d)$, $r = [\sigma_1^2, \sigma_2^2, ..., \sigma_D^2]^{\mathrm{T}}$, and $i = \mathrm{vec}(I)$. The virtual array steering matrix $\tilde{A}$ corresponds to the difference coarray of coprime array with more nominal sensors, where the increased DOF performance is guaranteed. From (9) and the equality

$$(\mathbf{\Phi}^* \otimes \mathbf{\Phi})(a^*(\theta_d) \otimes a(\theta_d)) = \mathbf{\Phi}^*a^*(\theta_d) \otimes \mathbf{\Phi}a(\theta_d), \tag{10}$$

z can be further expressed as

$$z = \breve{\mathbf{D}}r + \sigma_n^2\breve{i} \tag{11}$$

where $\breve{\mathbf{D}} = [\breve{d}(\theta_1), \breve{d}(\theta_2), ..., \breve{d}(\theta_D)] \in \mathbb{C}^{Q^2 \times D}$ is the compressed virtual array steering matrix with $\breve{d}(\theta_d) = \mathbf{\Phi}^*a^*(\theta_d) \otimes \mathbf{\Phi}a(\theta_d)$, and $\breve{i} = (\mathbf{\Phi}^* \otimes \mathbf{\Phi})i$. Therefore, the vector z behaves like an equivalent virtual array signal, whose corresponding array geometry is defined by the compressed virtual array steering matrix $\breve{\mathbf{D}}$.

After the compressed measurements have been derived to the virtual domain, we incorporated the sparsity-based framework [19] to achieve increased DOFs, where DOA estimation can be realised through sparse signal reconstruction. In particular, the optimisation problem of the proposed ID-COM method can be formulated by minimising the deviation between the derived equivalent virtual array signal z in (11) and its sparse version as

$$\min_{\bar{r}} \| \bar{r} \|_0 \quad \text{subject to} \quad \| z - \bar{\breve{\mathbf{D}}}\bar{r} - \sigma_n^2\breve{i} \|_2 < \epsilon, \tag{12}$$

where $\bar{\breve{\mathbf{D}}} = [\breve{d}(\bar{\theta}_1), \breve{d}(\bar{\theta}_2), ..., \breve{d}(\bar{\theta}_{\bar{D}})] \in \mathbb{C}^{Q^2 \times \bar{D}}$ is the sparse version of the compressed virtual array steering matrix with $\bar{D} \gg D$, and $\bar{r} = [\bar{\sigma}_1^2, \bar{\sigma}_2^2, ..., \bar{\sigma}_{\bar{D}}^2]^{\mathrm{T}}$ consists of the power of $\bar{D}$ sources, which are the collection of hypothetical directions over a predefined grid $\{\bar{\theta}_1, \bar{\theta}_2, ..., \bar{\theta}_{\bar{D}}\}$. Here, ϵ is a user-parameter to determine the reconstruction uncertainty bound.

The non-convex optimisation problem brought by the ℓ_0-norm can be solved by convex relaxation, such as least absolute shrinkage and selection operator [31]. In detail, the non-convex ℓ_0-norm can be replaced by the convex ℓ_1-norm. In so doing, the original non-convex optimisation problem (12) can be reformulated as a basis pursuit denoising problem [32]

$$\min_{\bar{r}, \sigma_n^2} \frac{1}{2}\| z - \breve{\mathbf{D}}\bar{r} - \sigma_n^2\breve{i} \|_2 + \xi\| \bar{r} \|_1, \tag{13}$$

where ξ is a regularisation parameter to balance the sparsity of the reconstructed spatial spectrum and the error of the ordinary least-squares cost function. The optimisation problem (13) is convex, and can be efficiently solved using the interior-point method software, such as CVX [33]. The DOA estimations can be obtained by searching for the peaks of the reconstructed sparse spatial spectrum.

The computational complexity of the proposed ID-COM method is $\mathcal{O}(\bar{D}Q^2)$. Compared with the sparsity-based method in [19] whose computational complexity is $\mathcal{O}(\bar{D}(M+N-1)^2)$, the proposed ID-COM method enjoys the computational efficiency due to the utilisation of compressive sensing on coprime array signal.
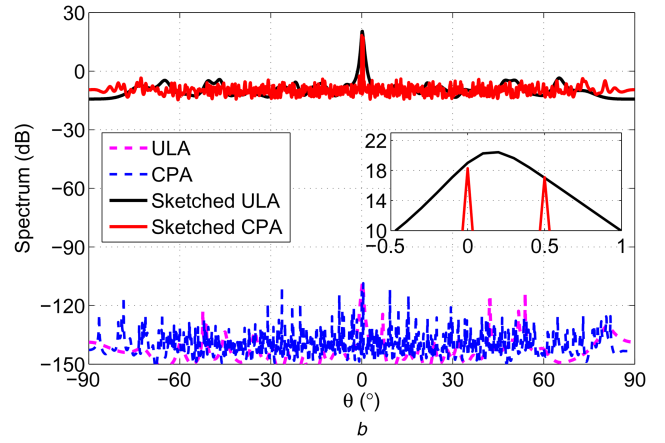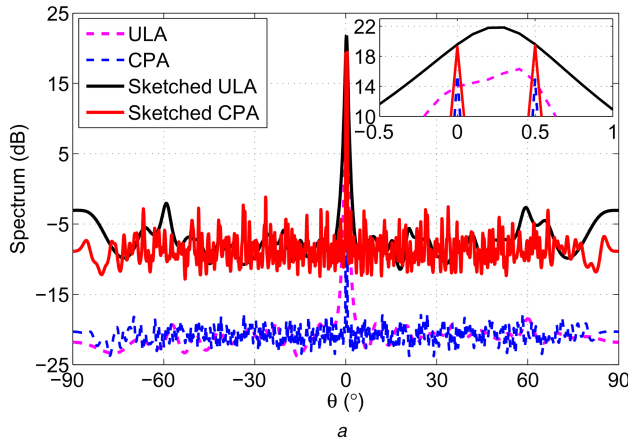
Fig. 2 *Capon spatial spectra of each DOA estimation method*
(a) Number of snapshots $K = 50$, *(b)* Number of snapshots $K = 20$

4 Numerical simulation results

In this section, we assess the performance of the proposed DOA estimation methods via numerical simulations and compare their performance with the state-of-the-art methods in the literature.

We first evaluate the high-resolution performance of the proposed HR-COM method. The pair of coprime integers is selected as $M = 18$ and $N = 19$. That is to say, the coprime array consists of $M + N - 1 = 36$ physical sensors with an array aperture of $324\lambda/2$. The number of measurements is set to be $Q = 8$, and the elements of 8×36 dimensional row orthogonal compressive sensing kernel $\boldsymbol{\Phi}$ are drawn from an independent and identically distributed random Gaussian distribution $\mathscr{CN}(0, 1/(M + N - 1))$. The hypothetical direction θ is within $[-90°, 90°]$ with the uniform sampling interval $\delta = 0.1°$.

The proposed HR-COM method described in (8) is compared with the Capon spatial spectra calculated from coprime array signals, ULA signals and compressed ULA signals [34]. For notational simplicity, their corresponding legends in the simulation figures are represented as Sketched CoPrime array (CPA), ULA, and sketched ULA, respectively. For fair comparison, the ULA also consists of 36 physical sensors, which has an array aperture of $35\lambda/2$. Meanwhile, the same random compressive sensing matrix $\boldsymbol{\Phi} \in \mathbb{C}^{8 \times 36}$ is applied for both the compressed coprime array signal and the compressed ULA signal. Hence, they have the same measurement size of 8×1, compared with the measurement size of 36×1 for both the coprime array signal and the ULA signal.

In Fig. 2, we assume that there are two closely-spaced independent sources impinging on the coprime array from the directions $0°$ and $0.5°$, and compare the Capon spatial spectra of the tested methods with respect to different snapshots. The signal-to-noise ratio (SNR) in each sensor is equal to 20 dB. In Fig. 2a, we set the number of snapshots $K = 50$, which is larger than the number of sensors. It is clear that the Capon spatial spectrum obtained from the CPA and sketched CPA can separate the two sources ($0°$ and $0.5°$). In contrast, benefiting from the large array aperture of the coprime array, the two sources are well separated by both the proposed HR-COM method using compressed coprime array signals and the Capon spatial spectrum using coprime array signals. In Fig. 2b, we consider the case with a very limited number of snapshots $K = 20$, which is less than the number of sensors. Because the sample covariance matrix $\hat{\boldsymbol{R}}_{xx} \in \mathbb{C}^{36 \times 36}$ is rank deficient in this case with insufficient number of snapshots, the Capon spatial spectra using ULA signals or coprime array signals fail to separate the two sources irrespective of the array geometry. Limited by the array aperture, the Capon spatial spectrum using compressed ULA signals can neither separate the two sources. However, the proposed HR-COM method using compressed coprime array signals can still effectively identify the two sources because the compressed sample covariance matrix $\hat{\boldsymbol{R}}_{yy} \in \mathbb{C}^{8 \times 8}$ calculated from 20 snapshots is full rank. Hence, the proposed HR-COM method can effectively identify the closely-spaced sources
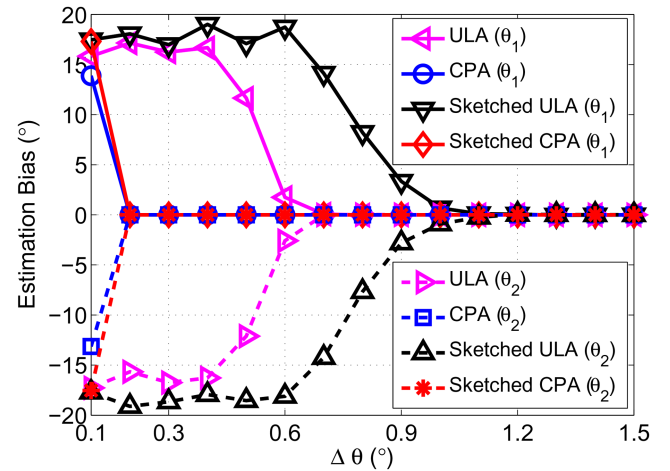

Fig. 3 *Estimation bias of each DOA estimation method versus angular separation*

even when the number of snapshots K is less than the number of physical sensors $M + N - 1$. Meanwhile, its computational complexity is much lower than those using the coprime array signals directly. This advantage is very useful for the practical applications when few number of snapshots are available, and the proposed HR-COM method is effective as long as the number of snapshots K is larger than the number of measurements Q, which contributes a full-rank condition for $\hat{\boldsymbol{R}}_{yy}$.

In Fig. 3, we compare the estimation bias of each DOA estimation method in terms of the angular separation with the condition that SNR = 20 dB and $K = 50$. For each scenario, 1,000 Monte–Carlo runs are performed. The two sources are assumed to have DOAs θ_1 and $\theta_1 + \Delta\theta$, respectively, where $\Delta\theta$ is the angular separation. It can be seen from Fig. 3 that the resolution of the proposed HR-COM method using compressed coprime array signals reaches the same $0.2°$ accuracy as the method that uses coprime array signals directly, while the resolutions of the methods using ULA signals and compressed ULA signals are $0.7°$ and $1.1°$, respectively. Therefore, the proposed HR-COM method outperforms the method using ULA signals in terms of the resolution of the estimated DOA. In addition, the proposed HR-COM method has a lower computational complexity than those using coprime array signals directly, while the high resolution offered by the coprime array can be effectively maintained.

Furthermore, we also evaluate the average root mean square error (RMSE) performance of each DOA estimation method obtained from 1,000 Monte–Carlo runs. The two incident sources are spaced $0.5°$ apart, namely, θ_r and $\theta_r + 0.5°$, where θ_r is randomly selected from Gaussian distribution $\mathcal{N}(0°, 1)$. Here, θ_r changes from run to run however it remains fixed from snapshot to snapshot. The SNR is set to be 10 dB when we vary the number of snapshots,
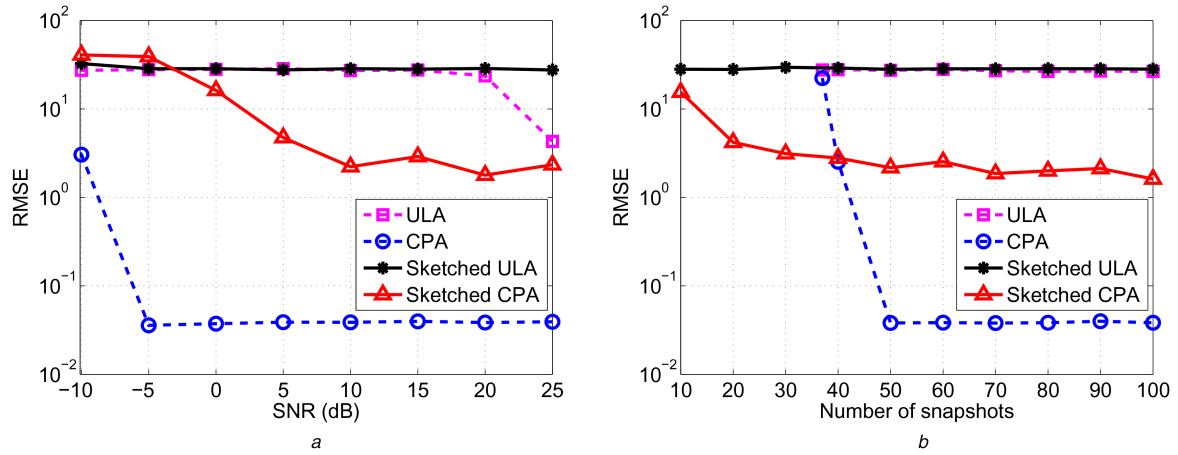
Fig. 4 *RMSE performance of each DOA estimation method*
(a) RMSE versus SNR with the number of snapshots $K = 50$, *(b)* RMSE versus number of snapshots with SNR = 10 dB
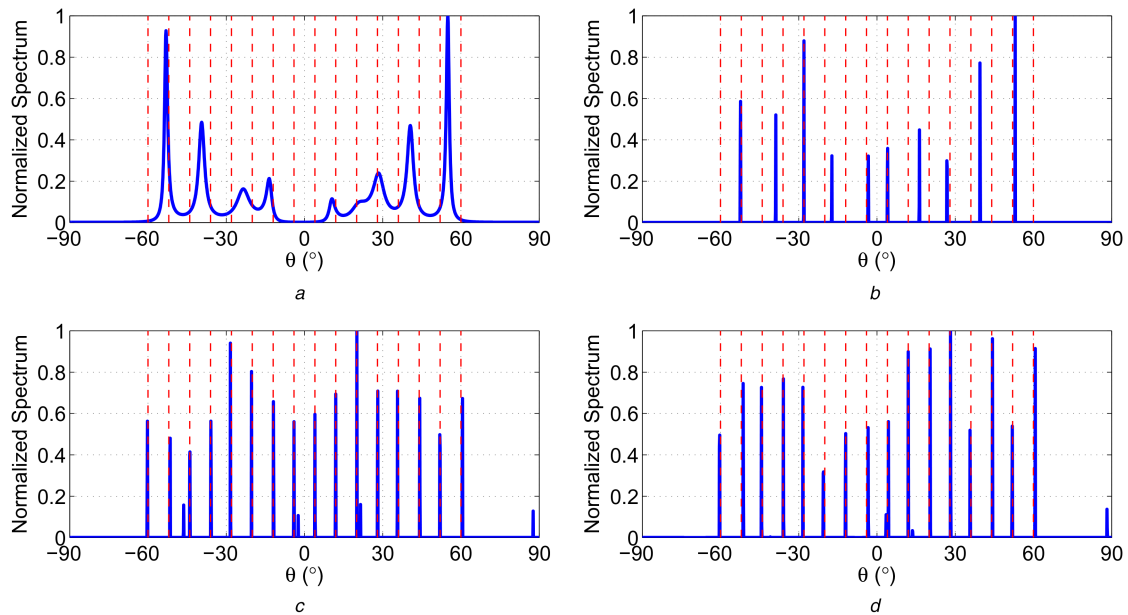

Fig. 5 *Normalised spectra of each DOA estimation method using coprime array*
(a) Coprime MUSIC method [15], *(b)* Covariance matrix sparse reconstruction method [18], *(c)* Sparsity-based method [19], *(d)* Proposed ID-COM method

whereas the number of snapshots is selected to be $K = 50$ when we vary the SNR. We can observe from Fig. 4*a* that the proposed method using compressed coprime array signals outperforms the method using compressed ULA signals [34] as well as the method using ULA signals when the SNR is larger than 0 dB. Although the method directly using coprime array signals yield lesser RMSE results, it requires a much higher computational complexity than the proposed method due to the computation of a high-dimensional covariance matrix. More importantly, the method using coprime array signals is not capable to perform DOA estimation when $K < M + N$ due to the rank-deficient problem. Nevertheless, the proposed algorithm using compressed coprime array signals is still effective even $K = 10$ according to the results illustrated in Fig. 4*b*, where the superiority of the proposed method is demonstrated.

We then evaluate the proposed ID-COM method depicted in (13) from the perspective of increased DOFs. Ten physical sensors are utilised to deploy the coprime array with coprime integers $M = 5$ and $N = 6$. Assuming that there are 16 incident sources with the directions uniformly distributed in $[-60°, 60°]$. The other parameters are the same as those used in the first simulation. Considering that the existing methods using ULA fail to identify all of the incident sources due to DOF limitation, here we compare the proposed ID-COM method with the methods using coprime array, including the coprime (multiple signal classification) MUSIC method [15], the covariance matrix sparse reconstruction method [18], and the sparsity-based method [19]. The

regularisation parameter ξ is selected as 0.25 for the optimisation problem (13).

The normalised spectra of each method are depicted in Fig. 5 with SNR = 10 dB and $K = 500$, where the red dashed lines denote the exact directions of incident sources. We can observe that the coprime MUSIC method and the covariance matrix sparse reconstruction method fail to identify all of the 16 sources. The reason lies in that both methods contain a spatial smoothing step, which requires a contiguous virtual array; hence, the maximum achievable number of DOFs is 10. By contrast, the sparsity-based DOA estimation method utilises all of the nominal sensors in the virtual array for sparse signal reconstruction. Since the derived virtual array achieves more nominal sensors than physical sensors, the spatial spectrum shown in Fig. 5*c* is capable to identify all of the 16 sources as expected. When it comes to the proposed ID-COM method, as shown in Fig. 5*d*, all the 16 sources can also be accurately identified by using the compressed coprime array signal $y(k)$ of size 8×1. Therefore, the compressed coprime array signal is capable to reserve the information of the original coprime array signal. Since the proposed method presents a similar spatial spectrum characteristic with the sparsity-based method in [19], the proposed ID-COM method enjoys a higher computational efficiency while maintains the advantage of DOFs offered by the coprime array.

5 Conclusion

In this paper, we focused on the newly emerged coprime array geometry for wireless communication systems and proposed two DOA estimation methods to effectively exploit the compressive sensing coprime array signal, namely, HR-COM method and ID-COM method. With a compressive sensing kernel, the coprime array signal is compressed to a lower-dimensional sketch through random projection, which leads to a reduced computational complexity without losing the original performance advantages for DOA estimation. To achieve high-resolution DOA estimation performance offered by the large array aperture of the coprime array, the proposed HR-COM method was designed to directly incorporate the compressed measurements. On the other hand, we also utilise the derived virtual signal of the compressed measurements for sparse signal reconstruction, and the ID-COM method was formulated to utilise the increased number of DOFs offered by the virtual array. Simulations results demonstrated the effectiveness of the proposed DOA estimation methods on computational complexity, resolution, and the number of DOFs.

6 Acknowledgment

7 References

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