A Novel Partial Relay Selection Method for Amplify-and-Forward Relay Systems

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***Abstract*— Although partial relay selection (PRS) for amplify-and-forward relay systems requires only the knowledge of source-relay (S-R) channels, in general, it incurs a significant performance loss. In cooperative systems with all single-antenna nodes, irrespective of the number of relays, the diversity order of PRS is limited to only one. This paper proposes a novel relay selection method for improving the performance of PRS scheme. In particular, as in the conventional PRS scheme, the relay that gives the best first-hop signal-to-noise ratio (SNR) is selected. However, this selection is made from only a subset of relays, for which the corresponding S-R and relay-destination (R-D) links are not in outage. An R-D link is considered to be in outage if its SNR is below the predefined threshold value of the end-to-end SNR plus some adjustable *margin*. The additional overhead required for implementing the proposed scheme is comparable to that of the conventional PRS method. For conciseness and better exposition of the proposed method, we limit our theoretical analysis to a system with two to three relays. The exact expressions of the end-to-end outage probability are derived and it is shown that full diversity order is achieved. Simulation results verify theoretical analysis and show that the proposed method significantly outperforms the conventional PRS method. Moreover, the results demonstrate that, for properly selected margin, the performance of the proposed method is very close or comparable to the method with full channel state information.**

***Index Terms*— Partial relay selection, amplify-and-forward relays, outage probability.**

I. Introduction

Cooperative communication [1] is an emerging technique that improves spatial diversity, coverage range and system throughput in wireless networks. A key component of cooperative communications is relaying in which a source takes help of user terminals in its coverage area to relay the source signal to the destination. Among different relay protocols [1], the amplify-and-forward (AF) protocol is attractive due to its simplicity. However, in systems with multiple relays, the feedback requirements and the overall implementation cost increase. Furthermore, the optimization of beamformers or space-time codes applied over the set of relays requires strict time and phase synchronization among them. In this context, *relay selection* is considered as an efficient approach for reducing signaling overhead and system design complexity while keeping intact the diversity gain. In [2]-[5], several relay selection schemes (known as opportunistic relaying (OR)) are proposed, which mainly require the instantaneous signal-to-noise ratio (SNR) of both source-relay (S-R) and relay-destination (R-D) links. Although [2] proposed distributed implementation of relay selection, such approach requires perfect synchronization among different nodes. This has motivated the authors of [6] to propose a centralized relay selection scheme with only the knowledge of the first-hop (i.e., S-R) channels. This scheme, known as *partial relay selection* (PRS), is attractive because it does not require global channel state information (CSI) at the central node and reduces the feedback requirements for acquiring CSI and maintaining synchronization. In [6]-[7] and [8], the performance of the PRS scheme is analyzed for CSI-assisted and fixed-gain AF relaying systems, respectively. However, in all these works ([6]-[8]), the PRS method incurs significant performance loss compared to the scheme with full CSI. In particular, irrespective of the number of relays, the diversity order of the PRS scheme is limited to only one. In our recent work [9], it is shown that the PRS scheme for multi-antenna cooperative systems with beamforming also suffers diversity loss for general antenna configurations.

This paper aims to improve the performance of conventional PRS at a cost of minimal increase in system implementation complexity. In particular, we propose a novel relay selection method which, as in the case of conventional PRS scheme, selects the relay that gives the best first-hop SNR. However, this selection is made from only a subset of relays, for which the corresponding S-R and R-D channels are not in outage condition. An S-R link is considered to be in outage if the corresponding link SNR is below the threshold value set for determining the outage event of the end-to-end transmission, whereas, an R-D link is in outage if its SNR is below the abovementioned threshold value plus some adjustable margin. A transmission protocol for implementing this selection method is proposed and its complexity is compared with the known PRS scheme. For the sake of clarity and conciseness, the theoretical analysis is presented for a system with two to three relays. We derive exact outage probability expressions and show that the proposed scheme achieves full diversity order. Note that, for making analysis concise, the theoretical results are obtained further assuming that both the S-R and R-D channels are identically distributed. However, it will be evident from the derivations that the analysis can be easily extended to the case with non-identically distributed channels. Moreover, simulation results show that the new scheme provides performance which is comparable to that of the scheme having full CSI [7].

The rest of the paper is organized as follows. The system model and the proposed relay selection method are presented in Section II. The outage probability expressions and the

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corresponding asymptotic performance are derived in Section III. In Section IV, the performance of the proposed method is compared with the conventional PRS and the method having full CSI. Section V concludes the paper.

Notations: $f_\alpha(x)$, $\Pr\{\cdot\}$, $\Pr\{x_1, x_2\}$, $\Pr\{[x_1 \cup x_2]\}$ and $\Pr\{x_1|x_2\}$ denote probability density function (PDF) of a continuous random variable α, the probability operator, the probability of the intersection between x_1 and x_2, the probability of the union of x_1 and x_2, and the probability of x_1 conditioned to x_2, respectively.

II. System model and Proposed Relay Selection

We study a cooperative network that consists of a source, a destination and R relay nodes, all equipped with single-antennas. The relays operate in a half-duplex mode and employ the AF protocol. All channels are assumed to be slowly time varying, i.e., the channel coherence time is much larger than the symbol/block duration. Let $h_{\mathrm{s},j}$ and $h_{j,\mathrm{d}}$ denote the complex channel coefficients between the source and the jth relay ($j = 1, \cdots, R$), and between the jth relay and the destination, respectively. Each node (source and relay) transmits with a given fixed power. Let P_{S} be the source power and $P_{\mathrm{R},j}$ the transmit power of the jth relay node. The path attenuations for the S-R and R-D channels are denoted by $c_{\mathrm{s},j}$ and $c_{j,\mathrm{d}}$, respectively. We assume Rayleigh fading environment, i.e., $h_{\mathrm{s},j}$ and $h_{j,\mathrm{d}}$ are assumed to be zero-mean circularly symmetric complex Gaussian (ZMCSCG) random variables with unit variance. The direct link between the source and destination does not exist since we consider that the destination is out of the coverage range of the source node.

A. Protocol description

In our proposed method, the source selects the relay node. As in the conventional PRS method, this selection is based on the instantaneous CSI of the S-R channels. However, the selection is made from only a subset of relays, for which the corresponding S-R and R-D links are not in outage. In order to implement this selection, we consider that the data transmission phase is followed by a training phase, in which a) the destination node broadcasts a training signal, b) the jth relay determines whether the SNR of its link with the destination is above a certain threshold value plus some margin (i.e., the link outage condition is checked), and c) only the relays that are not in outage (with reference to the R-D channels), as determined in the previous step, transmit training signals to the source node. The source node understands that the relays from which the source does not receive training signals have their corresponding R-D links in outage, and thus, such relay nodes are excluded from the selection process to be executed at the source. Among the relay nodes having their respective R-D links not in outage, the source further removes the relays, for which the S-R links are in outage, i.e., the corresponding link SNRs fall below the threshold value. Finally, the source selects one from the remaining relays that gives the maximum S-R SNR. It is important to highlight that the resulting end-to-end transmission from the source to the destination can still be subject to outage. To illustrate this fact, let us consider the case that the qth relay is selected, where $q \in \{1, R\}$. The SNR at the destination corresponding to the selected relay can be expressed as [11]

$$\gamma_q = \frac{g_q \alpha_{\mathrm{s},q} f_q \alpha_{q,\mathrm{d}}}{g_q \alpha_{\mathrm{s},q} + f_q \alpha_{q,\mathrm{d}} + 1} \leq \min(g_q \alpha_{\mathrm{s},q}, f_q \alpha_{q,\mathrm{d}}) \qquad (1)$$

where $\alpha_{\mathrm{s},q} = |h_{\mathrm{s},q}|^2$, $\alpha_{q,\mathrm{d}} = |h_{q,\mathrm{d}}|^2$, $g_q \triangleq \frac{P_{\mathrm{S}} c_{\mathrm{s},q}}{\sigma_q^2}$ and $f_q \triangleq \frac{P_{\mathrm{R},q} c_{q,\mathrm{d}}}{\sigma_{\mathrm{d}}^2}$ denote the average SNRs of the qth S-R and R-D links, respectively, and σ_q^2 and σ_{d}^2 are the corresponding noise powers at the qth relay and destination nodes. Since $h_{\mathrm{s},q}$ and $h_{q,\mathrm{d}}$ are ZMCSCG random variables, $\alpha_{\mathrm{s},q}$ and $\alpha_{q,\mathrm{d}}$ are exponentially distributed. Even if the SNRs of the S-R and R-D links corresponding to the qth relay are above a certain threshold value γ_{th} and above γ_{th} plus some margin $\epsilon > 0$, i.e., $\alpha_{\mathrm{s},q} \geq \frac{\gamma_{\mathrm{th}}}{g_q}$ and $\alpha_{q,\mathrm{d}} \geq \frac{\gamma_{\mathrm{th}}+\epsilon}{f_q}$, respectively, γ_q in (1) can still be smaller than γ_{th}. For example, consider that $\alpha_{\mathrm{s},q} = \frac{\gamma_{\mathrm{th}}}{g_q}$ and $\alpha_{q,\mathrm{d}} = \frac{\gamma_{\mathrm{th}}+\epsilon}{f_q}$. In this case, (1) reduces to

$$\gamma_q = \gamma_{\mathrm{th}} \frac{\gamma_{\mathrm{th}} + \epsilon}{2\gamma_{\mathrm{th}} + \epsilon + 1} \leq \gamma_{\mathrm{th}} \qquad (2)$$

where the equality holds for $\gamma_{\mathrm{th}} = 0$, which means that (2) shows that the outage can occur in the end-to-end transmission from the source to the destination. On the other hand, it is clear that such end-to-end transmission is certain to be in outage if all of the two-hop links from the source to the destination are found to be in outage. Although the latter case contributes to the overall outage probability at the destination as well, the source refrains from transmitting data since the outage is certain to occur. As such, system power consumption is reduced as no signal is transmitted. Finally, after the source determines the best relay, the training phase ends and the data transmission phase follows. The conventional two-phase single-relay transmission is then employed for data transmission using the selected relay.

The index q is selected from a set $\mathcal{A}$ which is defined as

$$\mathcal{A} = \left\{ j, \text{ s.t. } \left[\alpha_{\mathrm{s},j} \geq \frac{\gamma_{\mathrm{th}}}{g_j}, b_j = 1 \right]_{j=1}^{R} \right\} \qquad (3)$$

where b_j is a binary variable which takes the value of 1 if the R-D link corresponding to the jth relay is not in outage and 0, if the latter link is in outage. Mathematically, this can be expressed as

$$b_j = \begin{cases} 1, \text{ if } \alpha_{j,\mathrm{d}} \geq \frac{\gamma_{\mathrm{th}}+\epsilon}{f_j}, \\ 0, \text{ otherwise}, \end{cases} \qquad (4)$$

where $\epsilon > 0$ and needs to be properly selected for achieving the best performance. The selection rule is then given by

$$q = \arg\max_{j \in \mathcal{A}} \{\alpha_{\mathrm{s},j}\} \qquad (5)$$

which requires only the knowledge of S-R channels at the source since it implicitly has the knowledge of $\mathcal{A}$ due to the fact that, as mentioned before in the protocol description, only the relays having R-D links in non-outage condition send training signals to the source node. We can also define the set $\mathcal{E}$ which is the set of all j two-hop links, for which any of the S-R and R-D links are in outage. This set can be described as

$$\mathcal{E} = \left\{ j, \text{ s.t. } \left[\left(\alpha_{\mathrm{s},j} < \frac{\gamma_{\mathrm{th}}}{g_j} \right) \cup (b_j = 0) \right]_{j=1}^{R} \right\}. \qquad (6)$$

Note that the two-hop links or the corresponding relays of $\mathcal{E}$ are excluded from relay selection but the probability of the occurrence of $\mathcal{E}$ contributes to the end-to-end outage probability. We end this section with the following remark.

Remark 1: If no margin is used, i.e., $\epsilon = 0$, the selection method may result into the selection of the relay having its R-D SNR very close to $\gamma_{\rm th}$. This can lead to end-to-end outage events which can be prominent when the average SNR and number of relays increase, since, in these cases, the probability of more than one relay participating in relay selection increases. On the other hand, by taking $\epsilon > 0$, we may increase the occurrence of $\mathcal{E}$, although such probability decreases when the average SNR increases. However, in general, ϵ should be properly selected, for which, we pursue a numerical approach (cf. Section IV) as the analytical method is under research.

B. Complexity comparison between PRS and proposed method

In a conventional PRS scheme [6], the relay that gives the best first-hop SNR is selected. This means that the source node needs the knowledge of all S-R channels if the selection is performed at the source. It follows that an efficient way to make S-R channels available at the source node is through channel estimation in which the source estimates the S-R channels using the training signals transmitted by the relay nodes. This typically requires R training phases since the source is a single-antenna node and, thus, the relays need to transmit training signals sequentially. In our proposed method, however, the source node requires to estimate the S-R channels corresponding to only the relays that have their R-D links not in outage condition. This means that our scheme requires, in average, $\sum_{j=1}^{R} \Pr\{(b_j = 1)\}$ training phases for the estimation of S-R channels at the source. However, the proposed method requires an additional training phase in which the destination node transmits the training signal to the relay nodes which estimate the SNRs of R-D links and determine whether these links are in outage. As a result, the proposed method requires $1+\sum_{j=1}^{R} \Pr\{(b_j = 1)\}$ phases for training, which is comparable to the R training phases required in conventional PRS scheme. The additional effort required by the relay nodes for estimating the SNRs of R-D links in our case is equivalent to the computation of the minimum-mean-square estimates of the R-D channels [12].

III. Outage Probability Analysis

The outage probability at the destination is defined as the probability that the received SNR falls below a threshold value. However, the received SNR is conditional to the selected relay, which in turn depends on the state of $\mathcal{A}$. Let $\mathcal{O}$ denote the outage event at the destination. Using total probability law [13], the outage probability can be expressed as

$$P_{{\rm o},R} = \sum_{l=1}^{2^R-1} \Big[\Pr\{\mathcal{O}|\mathcal{A}_l\}\Pr\{\mathcal{A}_l\} \triangleq \Pr\{\mathcal{O},\mathcal{A}_l\}\Big] \tag{7}$$

where 2^R-1 is the total number of possible states for $\mathcal{A}$ and $\mathcal{A}_l$ denotes the corresponding lth state. For a system with an arbitrary R and having non-identically distributed channels, it is tedious to analyze $P_{{\rm o},R}$, since, all events $\mathcal{A}_l$ should be analyzed, where the statistics associated with each $\mathcal{A}_l$ can be different. This motivates us to focus our analysis to $R \leq 3$. The presented analytical approach, however, can be extended for $R > 3$. For $R = 3$, the possible states of $\mathcal{A}$ can be ordered as $\mathcal{A}_1 = \{1,2,3\}$, $\mathcal{A}_2 = \{1,2\}$, $\mathcal{A}_3 = \{1,3\}$, $\mathcal{A}_4 = \{2,3\}$, $\mathcal{A}_5 = \{1\}$, $\mathcal{A}_6 = \{2\}$ and $\mathcal{A}_7 = \{3\}$. Again with total probability law, $\Pr\{\mathcal{O},\mathcal{A}_l\}$ can be expressed as

$$\begin{aligned}\Pr\{\mathcal{O},\mathcal{A}_l\} = &\sum_{k=1}^{L_{\mathcal{A}_l}} \Pr\left\{[\mathcal{O},\mathcal{A}_l]\middle|\alpha_{{\rm s},\mathcal{A}_l(k)} \geq \max_{j\neq\mathcal{A}_l(k), j\in\mathcal{A}_l} \alpha_{{\rm s},j}\right\}\\ &\times\Pr\left\{\alpha_{{\rm s},\mathcal{A}_l(k)} \geq \max_{j\neq\mathcal{A}_l(k), j\in\mathcal{A}_l} \alpha_{{\rm s},j}\right\}\end{aligned} \tag{8}$$

where $L_{\mathcal{A}_l}$ is the cardinality of $\mathcal{A}_l$ and $\mathcal{A}_l(k)$ is the kth element of $\mathcal{A}_l$. In order to reduce the number of terms required for computation in (8) and (7), we further consider that all S-R and R-D channels are identically distributed, i.e., we assume that $g_j = g$, $f_j = f$, for all $j = 1, \cdots, R$. In this case, we get the same values of $\Pr\{\mathcal{O},\mathcal{A}_l\}$ for $l \in [5,6,7]$, for $l \in [2,3,4]$, and each term of the summation in (8) is equal for $l = 1$. Thus, using (7) and (8), and without loss of generality (w.l.o.g.) taking $\mathcal{O} = [\gamma_1 \leq \gamma_{\rm th}]$, the end-to-end outage probability for $R = 3$ can be expressed as

$$\begin{aligned}P_{{\rm o},3} = &3\Pr\{\gamma_1 \leq \gamma_{\rm th}|\mathcal{B}_1\}\Pr\{\mathcal{B}_1\} + 2\times 3\Pr\{\gamma_1 \leq \gamma_{\rm th}|\mathcal{B}_2\}\\ &\times\Pr\{\mathcal{B}_2\} + 3\Pr\{\gamma_1 \leq \gamma_{\rm th}|\mathcal{B}_3\}\Pr\{\mathcal{B}_3\} + \Pr\{\mathcal{E}\}\end{aligned} \tag{9}$$

where the sets $\mathcal{B}_1$, $\mathcal{B}_2$, $\mathcal{B}_3$ are disjoint, and are defined as

$$\begin{aligned}\mathcal{B}_1 = &\left\{\alpha_{{\rm s},1} \geq \max(\alpha_{{\rm s},2},\alpha_{{\rm s},3}), [\alpha_{{\rm s},j}]_{j=1}^{3} \geq \frac{\gamma_{\rm th}}{g}, [b_j = 1]_{j=1}^{3}\right\}\\ \mathcal{B}_2 = &\left\{\alpha_{{\rm s},1} \geq \alpha_{{\rm s},2}, [\alpha_{{\rm s},j}]_{j=1}^{2} \geq \frac{\gamma_{\rm th}}{g}, [b_j = 1]_{j=1}^{2},\right.\\ &\left.\left[\alpha_{{\rm s},3} < \frac{\gamma_{\rm th}}{g} \cup b_3 = 0\right]\right\}\\ \mathcal{B}_3 = &\left\{\alpha_{{\rm s},1} \geq \frac{\gamma_{\rm th}}{g}, b_1 = 1, \left[\alpha_{{\rm s},2} < \frac{\gamma_{\rm th}}{g} \cup b_2 = 0\right],\right.\\ &\left.\left[\alpha_{{\rm s},3} < \frac{\gamma_{\rm th}}{g} \cup b_3 = 0\right]\right\}.\end{aligned} \tag{10}$$

We define $t_1 \triangleq \Pr\{\gamma_1 \leq \gamma_{\rm th}|\mathcal{B}_1\}\Pr\{\mathcal{B}_1\}$. Since γ_1 is independent of b_2 and b_3, we can write t_1 as

$$\begin{aligned}t_1 = &\Pr\left\{\gamma_1 \leq \gamma_{\rm th}\middle|\left\{\alpha_{{\rm s},1} \geq \max(\alpha_{{\rm s},2},\alpha_{{\rm s},3}), b_1 = 1,\right.\right.\\ &\left.\left.\{\alpha_{{\rm s},j}\}_{j=1}^{3} \geq \frac{\gamma_{\rm th}}{g}\right\}\right\}\Pr\{\mathcal{B}_1\}\\ = &\Pr\left\{\gamma_1 \leq \gamma_{\rm th}\middle|\left\{\alpha_{{\rm s},1} \geq \max(\alpha_{{\rm s},2},\alpha_{{\rm s},3}), b_1 = 1,\right.\right.\\ &\left.\left.\{\alpha_{{\rm s},j}\}_{j=1}^{3} \geq \frac{\gamma_{\rm th}}{g}\right\}\right\}\Pr\left\{\alpha_{{\rm s},1} \geq \max(\alpha_{{\rm s},2},\alpha_{{\rm s},3}), b_1 = 1,\right.\\ &\left.\{\alpha_{{\rm s},j}\}_{j=1}^{3} \geq \frac{\gamma_{\rm th}}{g}\right\}\Pr\{b_2 = 1\}\Pr\{b_3 = 1\}\\ = &\Pr\left\{\gamma_1 \leq \gamma_{\rm th}, \alpha_{{\rm s},1} \geq \max(\alpha_{{\rm s},2},\alpha_{{\rm s},3}), b_1 = 1,\right.\\ &\left.\{\alpha_{{\rm s},j}\}_{j=1}^{3} \geq \frac{\gamma_{\rm th}}{g}\right\}\Pr\{b_2 = 1\}\Pr\{b_3 = 1\}\end{aligned} \tag{11}$$

where the second step is due to the fact that $[b_2=1,b_3=1]$ and $\left[\alpha_{\mathrm{s},1}\geq\max(\alpha_{\mathrm{s},2},\alpha_{\mathrm{s},3}),b_1=1,\left\{\alpha_{\mathrm{s},j}\geq\frac{\gamma_{\mathrm{th}}}{g}\right\}_{j=1}^{3}\right]$ are statistically independent and the third step is due to the definition of conditional probability [13]. Since $b_j=1$ implies that $\alpha_{j,\mathrm{d}}\geq\frac{\gamma_{\mathrm{th}}+\epsilon}{f}(j=2,3)$, we have $\Pr\{b_2=1\}=\Pr\{b_3=1\}=\mathrm{e}^{-\frac{(\gamma_{\mathrm{th}}+\epsilon)}{f}}$. Let the first probability term of t_1 in (11) be denoted by $\tilde{t}_1$. As shown in Appendix A, $\tilde{t}_1$ can be expressed as

$$\tilde{t}_1=\mathrm{e}^{-\gamma_{\mathrm{th}}\left(\frac{3}{g}+\frac{1}{f}\right)}\left\{\frac{\mathrm{e}^{-\frac{\epsilon}{f}}}{3}+s_2-s_1-\frac{1}{3}s_3\right\} \tag{12}$$

where

$$s_m=\mathrm{e}^{-\frac{\epsilon}{f}}+\sum_{k=1}^{\infty}(-m\gamma_{\mathrm{th}}(\gamma_{\mathrm{th}}+1))^k\frac{g^{-k}}{k!}(\epsilon f)^{-\frac{k}{2}} \times\mathrm{e}^{-\frac{\epsilon}{2f}}W_{-\frac{k}{2},\frac{1-k}{2}}\left(\frac{\epsilon}{f}\right),m=[1,2,3], \tag{13}$$

and $W_{\lambda,\mu}(\ )$ is the Whittaker hypergeometric function [10]. Applying eq. (3.324.1) of [10] in (31) of Appendix A, for the case $\epsilon=0$, s_m can be expressed as

$$s_m^{\epsilon=0}=2\sqrt{\frac{m\gamma_{\mathrm{th}}(\gamma_{\mathrm{th}}+1)}{gf}}K_1\left(2\sqrt{\frac{m\gamma_{\mathrm{th}}(\gamma_{\mathrm{th}}+1)}{gf}}\right) \tag{14}$$

where $K_1(.)$ is the modified Bessel function of the second type with order 1. We define $\tilde{t}_2\triangleq\Pr\{\gamma_1\leq\gamma_{\mathrm{th}}|\mathcal{B}_2\}\Pr\{\mathcal{B}_2\}$. Since γ_1 is independent of $[b_2=1]$ and $[\alpha_{\mathrm{s},3}<\frac{\gamma_{\mathrm{th}}}{g}\cup b_3=0]$, we can re-express $\tilde{t}_2$ as

$$\begin{aligned}\tilde{t}_2=&\Pr\left\{\gamma_1\leq\gamma_{\mathrm{th}}\middle|\left\{\alpha_{\mathrm{s},1}\geq\alpha_{\mathrm{s},2},b_1=1,\right.\right.\\&\left.\left.\{\alpha_{\mathrm{s},j}\}_{j=1}^{2}\geq\frac{\gamma_{\mathrm{th}}}{g}\right\}\right\}\Pr\{\mathcal{B}_2\}\\=&\Pr\left\{\gamma_1\leq\gamma_{\mathrm{th}},\alpha_{\mathrm{s},1}\geq\alpha_{\mathrm{s},2},b_1=1,\{\alpha_{\mathrm{s},j}\}_{j=1}^{2}\geq\frac{\gamma_{\mathrm{th}}}{g}\right\}\\&\times\Pr\{b_2=1\}\Pr\left\{\alpha_{\mathrm{s},3}<\frac{\gamma_{\mathrm{th}}}{g}\cup b_3=0\right\}.\end{aligned} \tag{15}$$

For convenience, let the first part of $\tilde{t}_2$ be defined as $\tilde{t}_{2,1}\triangleq\Pr\left\{\gamma_1\leq\gamma_{\mathrm{th}},\alpha_{\mathrm{s},1}\geq\alpha_{\mathrm{s},2},b_1=1,\{\alpha_{\mathrm{s},j}\}_{j=1}^{2}\geq\frac{\gamma_{\mathrm{th}}}{g}\right\}$. Note that the differences between $\tilde{t}_{2,1}$ and $\tilde{t}_1$ lie only in the facts that the event $\alpha_{\mathrm{s},1}\geq\max(\alpha_{\mathrm{s},2},\alpha_{\mathrm{s},3})$ of $\tilde{t}_1$ takes the form $\alpha_{\mathrm{s},1}\geq\alpha_{\mathrm{s},2}$ in $\tilde{t}_{2,1}$ and the event $\alpha_{\mathrm{s},3}\geq\frac{\gamma_{\mathrm{th}}}{g}$ of $\tilde{t}_1$ does not appear in $\tilde{t}_{2,1}$. Thus, using the methodology of Appendix A, $\tilde{t}_{2,1}$ can be readily derived. The expression for $\tilde{t}_{2,1}$ is then given by

$$\tilde{t}_{2,1}=\mathrm{e}^{-\frac{\gamma_{\mathrm{th}}}{f}}\mathrm{e}^{-2\frac{\gamma_{\mathrm{th}}}{g}}\left\{\frac{1}{2}\mathrm{e}^{-\frac{\epsilon}{f}}+\frac{1}{2}s_2-s_1\right\} \tag{16}$$

Let the second part of $\tilde{t}_2$ be defined as $\tilde{t}_{2,2}\triangleq\Pr\{b_2=1\}\Pr\left\{\alpha_{\mathrm{s},3}<\frac{\gamma_{\mathrm{th}}}{g}\cup b_3=0\right\}$. Applying the fact that $\Pr\{x_1\cup x_2\}$=$\Pr\{x_1\}+\Pr\{x_2\}-\Pr\{x_1\}\Pr\{x_2\}$ for statistically independent x_1 and x_2, noting that $\Pr\{b_2=1\}=\Pr\left\{\alpha_{2,\mathrm{d}}\geq\frac{\gamma_{\mathrm{th}}}{f}+\epsilon\right\}$, and $\alpha_{\mathrm{s},3}$, $\alpha_{2,\mathrm{d}}$ and $\alpha_{3,\mathrm{d}}$ are exponentially distributed, we readily get

$$\tilde{t}_{2,2}\ =\ \mathrm{e}^{-\frac{\gamma_{\mathrm{th}}+\epsilon}{f}}\left[1-\mathrm{e}^{-\frac{\gamma_{\mathrm{th}}}{g}}\mathrm{e}^{-\frac{\gamma_{\mathrm{th}}+\epsilon}{f}}\right]. \tag{17}$$

Thus, $\tilde{t}_2=\tilde{t}_{2,1}\tilde{t}_{2,2}$ is obtained. We define $\tilde{t}_3\triangleq\Pr\{\gamma_1\leq\gamma_{\mathrm{th}}|\mathcal{B}_3\}\Pr\{\mathcal{B}_3\}$. Since γ_1 is independent of the events $\left[\alpha_{\mathrm{s},j}<\frac{\gamma_{\mathrm{th}}}{g}\cup b_j=0\right]_{j=2}^{3}$, we can re-express $\tilde{t}_3$ as

$$\begin{aligned}\tilde{t}_3=&\Pr\left\{\gamma_1\leq\gamma_{\mathrm{th}}\middle|\left[b_1=1,\alpha_{\mathrm{s},1}\geq\frac{\gamma_{\mathrm{th}}}{g}\right]\right\}\Pr\{\mathcal{B}_3\}\\=&\underbrace{\Pr\left\{\gamma_1\leq\gamma_{\mathrm{th}},b_1=1,\alpha_{\mathrm{s},1}\geq\frac{\gamma_{\mathrm{th}}}{g}]\right\}}_{\tilde{t}_{3,1}}\\&\times\underbrace{\Pr\left\{\left[\alpha_{\mathrm{s},j}\leq\frac{\gamma_{\mathrm{th}}}{g}\cup b_j=0\right]_{j=2}^{3}\right\}}_{\tilde{t}_{3,2}}\end{aligned} \tag{18}$$

which follows due to the statistical independence of $\left[b_1=1,\alpha_{\mathrm{s},1}\geq\frac{\gamma_{\mathrm{th}}}{g}\right]$ and $\left[\alpha_{\mathrm{s},j}\leq\frac{\gamma_{\mathrm{th}}}{g}\cup b_j=0\right]_{j=2}^{3}$ and the definition of conditional probability. Since $b_1=1$ implies that $\alpha_{1,\mathrm{d}}\geq\frac{\gamma_{\mathrm{th}}+\epsilon}{f}$, $\tilde{t}_{3,1}$ can be expressed as

$$\begin{aligned}\tilde{t}_{3,1}=&\int_{y=\frac{\gamma_{\mathrm{th}}+\epsilon}{f}}^{\infty}\Pr\left\{\alpha_{\mathrm{s},1}\leq\frac{\gamma_{\mathrm{th}}(fy+1)}{gfy-g\gamma_{\mathrm{th}}},\right.\\&\left.\alpha_{\mathrm{s},1}\geq\frac{\gamma_{\mathrm{th}}}{g}\right\}f_{\alpha_{1,\mathrm{d}}}(y)dy.\end{aligned} \tag{19}$$

Using similar steps as in Appendix A, $\tilde{t}_{3,1}$ can be given by

$$\tilde{t}_{3,1}=\mathrm{e}^{-\gamma_{\mathrm{th}}\left(\frac{1}{g}+\frac{1}{f}\right)}\left(\mathrm{e}^{-\frac{\epsilon}{f}}-s_1\right), \tag{20}$$

where s_1 is given in (13). Similarly, $\tilde{t}_{3,2}$ can be written as

$$\tilde{t}_{3,2}=\left[1-\mathrm{e}^{-\frac{\gamma_{\mathrm{th}}}{g}}\mathrm{e}^{-\frac{\gamma_{\mathrm{th}}+\epsilon}{f}}\right]^2. \tag{21}$$

The probability of the occurrence of $\mathcal{E}$ is given by

$$\begin{aligned}\Pr\{\mathcal{E}\}\ =&\ \Pr\left\{\left[[\alpha_{\mathrm{s},j}\leq\frac{\gamma_{\mathrm{th}}}{g}]\cup[\alpha_{j,\mathrm{d}}\leq\frac{\gamma_{\mathrm{th}}+\epsilon}{f}]\right]_{j=1}^{3}\right\}\\=&\ \left[1-\mathrm{e}^{-\frac{\epsilon}{f}}\mathrm{e}^{-\gamma_{\mathrm{th}}\left(\frac{1}{g}+\frac{1}{f}\right)}\right]^3.\end{aligned} \tag{22}$$

The outage probability (9) is then expressed as

$$\begin{aligned}P_{\mathrm{o},3}=&3\tilde{t}_1\mathrm{e}^{-2\left(\frac{\gamma_{\mathrm{th}}+\epsilon}{f}\right)}+6\tilde{t}_{2,1}\tilde{t}_{2,2}+3\tilde{t}_{3,1}\tilde{t}_{3,2}+\Pr\{\mathcal{E}\}\\=&3\mathrm{e}^{-2\gamma_{\mathrm{th}}\left(\frac{1}{g}+\frac{1}{f}\right)}\left\{\left\{\frac{1}{3}\mathrm{e}^{-\frac{\epsilon}{f}}+s_2-s_1-\frac{1}{3}s_3\right\}\mathrm{e}^{-\frac{2\epsilon}{f}}\right.\\&\times\mathrm{e}^{-\gamma_{\mathrm{th}}\left(\frac{1}{g}+\frac{1}{f}\right)}+2\mathrm{e}^{-\frac{\epsilon}{f}}\left[0.5s_2-s_1+0.5\mathrm{e}^{-\frac{\epsilon}{f}}\right]\\&\left.\times\left[1-\mathrm{e}^{-\gamma_{\mathrm{th}}\left(\frac{1}{g}+\frac{1}{f}\right)}\mathrm{e}^{-\frac{\epsilon}{f}}\right]\right\}+\left[1-\mathrm{e}^{-\gamma_{\mathrm{th}}\left(\frac{1}{g}+\frac{1}{f}\right)}\mathrm{e}^{-\frac{\epsilon}{f}}\right]^2\\&\times\left\{3\left(\mathrm{e}^{-\frac{\epsilon}{f}}-s_1\right)\mathrm{e}^{-\gamma_{\mathrm{th}}\left(\frac{1}{g}+\frac{1}{f}\right)}+1\right.\\&\left.-\mathrm{e}^{-\gamma_{\mathrm{th}}\left(\frac{1}{g}+\frac{1}{f}\right)}\mathrm{e}^{-\frac{\epsilon}{f}}\right\},\end{aligned} \tag{23}$$

which can be computed efficiently, since s_1, s_2, s_3 (see (13)) converge after a few terms, especially for medium to high values of both g and f. After these derivations for $R=3$, the outage probability expression for $R=2$ is readily given by $P_{\mathrm{o},2}=2\tilde{t}_{2,1}\mathrm{e}^{-\frac{\gamma_{\mathrm{th}}+\epsilon}{f}}+2\tilde{t}_{3,1}\tilde{t}_{3,2}^{1/2}+\Pr\{\mathcal{E}\}^{2/3}$.

Asymptotic Analysis: As shown in Appendix B, for high SNR (i.e., $g, f \rightarrow \infty$), (23) can be first approximated as

$$\begin{aligned} P_{\mathrm{o},3} \approx\, & 3\mathrm{e}^{-2\gamma_{\mathrm{th}}\left(\frac{1}{g}+\frac{1}{f}\right)}(\gamma_{\mathrm{th}}(\gamma_{\mathrm{th}}+1))^2\epsilon^{-1}\mathrm{e}^{-\frac{\epsilon}{2f}}g^{-2}f^{-1} \\ & \times\left(\frac{1}{3}\mathrm{e}^{-\gamma_{\mathrm{th}}\left(\frac{1}{g}+\frac{1}{f}\right)}g^{-1}(\epsilon f)^{-\frac{1}{2}}\mathrm{e}^{-\frac{2\epsilon}{f}}W_{-\frac{3}{2},-1}\left(\frac{\epsilon}{f}\right)+\mathrm{e}^{-\frac{\epsilon}{f}}\right. \\ & \left.\times\left[1-\mathrm{e}^{-\frac{\epsilon}{f}}\mathrm{e}^{-\gamma_{\mathrm{th}}\left(\frac{1}{g}+\frac{1}{f}\right)}\right]W_{-1,-\frac{1}{2}}\left(\frac{\epsilon}{f}\right)\right)+\left[1-\mathrm{e}^{-\frac{\epsilon}{f}}\right. \\ & \left.\times\mathrm{e}^{-\gamma_{\mathrm{th}}\left(\frac{1}{g}+\frac{1}{f}\right)}\right]^2\left\{3\mathrm{e}^{-\gamma_{\mathrm{th}}\left(\frac{1}{g}+\frac{1}{f}\right)}\gamma_{\mathrm{th}}(\gamma_{\mathrm{th}}+1)(\epsilon f)^{-1/2}\right. \\ & \left.\times\mathrm{e}^{-\frac{\epsilon}{2f}}g^{-1}W_{-\frac{1}{2},0}\left(\frac{\epsilon}{f}\right)+1-\mathrm{e}^{-\frac{\epsilon}{f}}\mathrm{e}^{-\gamma_{\mathrm{th}}\left(\frac{1}{g}+\frac{1}{f}\right)}\right\}. \quad (24) \end{aligned}$$

W.l.o.g., we assume that $g = \bar{\gamma}$ and $f = \eta\bar{\gamma}$, where $\eta > 0$. Then, after analyzing (24) and the asymptotic expressions for $W_{-1,-\frac{1}{2}}\left(\frac{\epsilon}{f}\right)$, $W_{-\frac{1}{2},0}\left(\frac{\epsilon}{f}\right)$, and $W_{-\frac{3}{2},-1}\left(\frac{\epsilon}{f}\right)$ [14] as $\frac{\epsilon}{f} \rightarrow 0$ (i.e., $f \rightarrow \infty$ for a given ϵ), we find that the term with the lowest negative exponent of $\bar{\gamma}$ is $\left[1-\mathrm{e}^{-\frac{\epsilon}{\eta\bar{\gamma}}}\mathrm{e}^{-\gamma_{\mathrm{th}}\left(\frac{1}{\bar{\gamma}}+\frac{1}{\eta\bar{\gamma}}\right)}\right]^3$. Using the fact that $\mathrm{e}^{-x} \approx 1-x$ (for small x), this term can be approximated by $(\eta\bar{\gamma})^{-3}[\gamma_{\mathrm{th}}(1+\eta)(1-\frac{\epsilon}{\eta\bar{\gamma}})+\epsilon]^3$. This means that the diversity order of 3 is achieved.

IV. Numerical Results

In this section, we provide Monte Carlo simulations to assess the accuracy of the analytical expressions for the outage probability. We also compare the performance of our proposed method with the conventional PRS [6] and opportunistic relay selection based on full CSI (F-CSI) [7]. In all examples, we take $\gamma_{\mathrm{th}} = 3$ dB, $\sigma_j^2 = \sigma_{\mathrm{d}}^2 = \sigma_n^2$, $P_{\mathrm{S}} = P_{\mathrm{R},j} = P$, and $c_{j,\mathrm{d}} = c_{\mathrm{s},j} = c$, $\forall j$. We vary the average SNR defined as $\mathcal{SNR} \triangleq \frac{cP}{\sigma_n^2}$ by keeping $\sigma_n^2 = 1$. All channel coefficients are taken to be ZMCSCG random variables with unit variance. The outage probabilities as a function of $\mathcal{SNR}$ are shown in Fig. 1 for different R and selection methods. In this figure, we take $\epsilon = 0.08\gamma_{\mathrm{th}}$ for $R = 2$ and $\epsilon = 0.1\gamma_{\mathrm{th}}$ for $R = 3$. Fig. 1 demonstrates that there is a fine agreement between the theoretical and simulation results of the proposed scheme, which verifies the accuracy of the derived outage probability expressions. Moreover, the proposed method achieves full diversity, significantly outperforms conventional PRS scheme and provides performance which is very close/comparable to that of F-CSI method for $R = 2/R = 3$. Fig. 2 displays theoretical outage probability for the proposed method for different ϵ and $R = 3$. It can be observed from Fig. 2 that the performance degrades for both low ($\epsilon = 0$) or high ($\epsilon = \gamma_{\mathrm{th}}$) values of ϵ. In Fig. 3, we compare the simulated outage probability of our scheme for $R = 4$ (with $\epsilon = 0.25\gamma_{\mathrm{th}}$ for $\mathcal{SNR}$ <= 16 dB and $\epsilon = 0.15\gamma_{\mathrm{th}}$ for $\mathcal{SNR} = 20$ dB) with the theoretical outage probabilities of PRS and the F-CSI method. As in Fig. 1, the performance of our method is significantly better than that of the PRS scheme and comparable to that of the method with F-CSI. The fact that this performance gain over PRS scheme is achieved with a minimal increase in system complexity makes our proposed method attractive for practical systems.

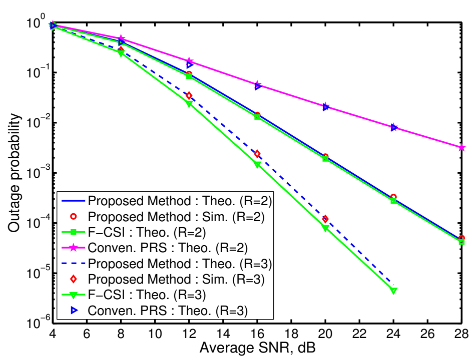

Fig. 1. Outage probability versus average SNR.

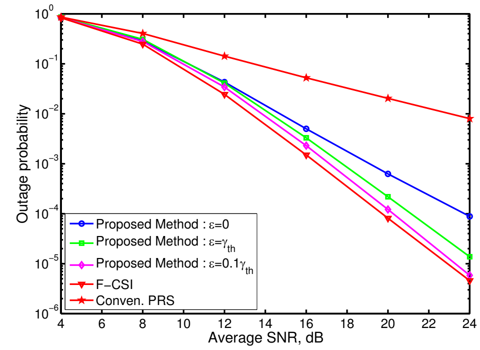

Fig. 2. Effect of ϵ on the outage probability (Theoretical for $R = 3$).

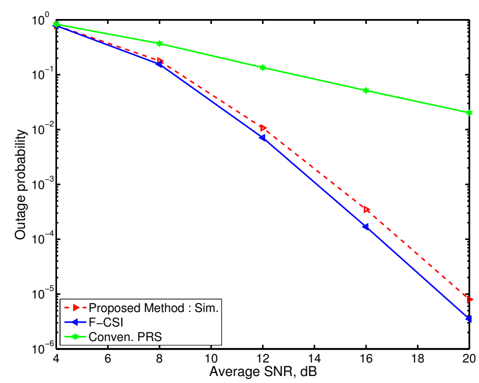

Fig. 3. Outage probability versus average SNR for $R = 4$.

V. Conclusions

We have proposed a novel method which selects the relay with the best first-hop SNR as in the case of conventional PRS scheme. However, this selection is made from only a subset of relays with both the S-R and R-D links in non-outage condition. For $R <= 3$, we derived outage probability expressions and showed that the proposed method achieves full diversity. The results show that the new method significantly outperforms PRS and provides performance that is comparable to that of the method with full CSI.

Appendix A

Substituting (1) into $\gamma_1 \leq \gamma_{\mathrm{th}}$ and integrating over the PDF of $\alpha_{1,\mathrm{d}}$, $\tilde{t}_1$ can be expressed as

$$\begin{aligned} \tilde{t}_1 = \int_{y=\frac{\gamma_{\mathrm{th}}+\epsilon}{f}}^{\infty} \Pr\Big\{ & \alpha_{\mathrm{s},1} \leq \frac{\gamma_{\mathrm{th}}(fy+1)}{gfy-g\gamma_{\mathrm{th}}}, \{\alpha_{\mathrm{s},j}\}_{j=1}^{3} \geq \frac{\gamma_{\mathrm{th}}}{g}, \\ & \alpha_{\mathrm{s},1} \geq \alpha_{\mathrm{s},2}, \alpha_{\mathrm{s},1} \geq \alpha_{\mathrm{s},3} \Big\} f_{\alpha_{1,\mathrm{d}}}(y)\, dy \qquad (25) \end{aligned}$$

where $f_{\alpha_{1,\mathrm{d}}}(y)$ is the PDF of $\alpha_{1,\mathrm{d}}$ and the event $\alpha_{\mathrm{s},1} \geq \max\{\alpha_{\mathrm{s},j}\}_{j=2}^{3}$ is written as $[\alpha_{\mathrm{s},1} \geq \alpha_{\mathrm{s},2}, \alpha_{\mathrm{s},1} \geq \alpha_{\mathrm{s},3}]$. Since $\{\alpha_{\mathrm{s},j}\}_{j=1}^{3} \geq \frac{\gamma_{\mathrm{th}}}{g}$, (25) can be further expressed as

$$\tilde{t}_1 = \int_{v=\frac{\gamma_{\mathrm{th}}}{g}}^{\bar{v}} \int_{u=\frac{\gamma_{\mathrm{th}}}{g}}^{\bar{u}} \int_{y=\frac{\gamma_{\mathrm{th}}+\epsilon}{f}}^{\infty} \Pr\Big\{\alpha_{\mathrm{s},1} \leq \frac{\gamma_{\mathrm{th}}(fy+1)}{gfy-g\gamma_{\mathrm{th}}}, \alpha_{\mathrm{s},1} \geq u, \alpha_{\mathrm{s},1} \geq v\Big\} f_{\alpha_{\mathrm{s},2}}(u)du f_{\alpha_{\mathrm{s},3}}(v)dv f_{\alpha_{1,\mathrm{d}}}(y)dy \quad (26)$$

where $f_{\alpha_{\mathrm{s},2}}(u)$, $f_{\alpha_{\mathrm{s},3}}(v)$ are the PDFs of $\alpha_{\mathrm{s},2}$ and $\alpha_{\mathrm{s},3}$, respectively, whereas, the respective upper limits of the variable u and v are $\bar{u}$ and $\bar{v}$. It is clear that the joint probability in (26) is not zero only for $\bar{u} = \bar{v} = \frac{\gamma_{\mathrm{th}}(fy+1)}{gfy-g\gamma_{\mathrm{th}}} \triangleq u_{\mathrm{yu}}$. Note that $\Pr\{\alpha_{\mathrm{s},1} \geq u, \alpha_{\mathrm{s},1} \geq v\}$ takes the following values

$$\Pr\{\alpha_{\mathrm{s},1} \geq u, \alpha_{\mathrm{s},1} \geq v\} = \begin{cases} \Pr\{\alpha_{\mathrm{s},1} \geq v\}, & \text{for } v \geq u \\ \Pr\{\alpha_{\mathrm{s},1} \geq u\}, & \text{for } u \geq v \end{cases}. \quad (27)$$

Due to (27), $\tilde{t}_1$ in (26) can be expressed as $\tilde{t}_1 = I_1 + I_2$, where

$$I_1 = \int_{y=\frac{\gamma_{\mathrm{th}}+\epsilon}{f}}^{\infty} \int_{u=\frac{\gamma_{\mathrm{th}}}{g}}^{u_{\mathrm{yu}}} \int_{v=u}^{u_{\mathrm{yu}}} \Pr\Big\{\alpha_{\mathrm{s},1} \leq \frac{\gamma_{\mathrm{th}}(fy+1)}{gfy-g\gamma_{\mathrm{th}}}, \alpha_{\mathrm{s},1} \geq v\Big\} f_{\alpha_{\mathrm{s},3}}(v)dv f_{\alpha_{\mathrm{s},2}}(u)du f_{\alpha_{1,\mathrm{d}}}(y)dy,$$

$$I_2 = \int_{y=\frac{\gamma_{\mathrm{th}}+\epsilon}{f}}^{\infty} \int_{v=\frac{\gamma_{\mathrm{th}}}{g}}^{u_{\mathrm{yu}}} \int_{u=v}^{u_{\mathrm{yu}}} \Pr\Big\{\alpha_{\mathrm{s},1} \leq \frac{\gamma_{\mathrm{th}}(fy+1)}{gfy-g\gamma_{\mathrm{th}}}, \alpha_{\mathrm{s},1} \geq u\Big\} f_{\alpha_{\mathrm{s},2}}(u)du f_{\alpha_{\mathrm{s},3}}(v)dv f_{\alpha_{1,\mathrm{d}}}(y)dy. \quad (28)$$

Since $\{\alpha_{\mathrm{s},j}\}_{j=1}^{3}$ are identically distributed, it is clear from the limits of the integrals I_1 and I_2 that $I_1 = I_2$. Thus, it is sufficient to solve one of the integrals in (29). With the variable substitution $\bar{y} = y - \frac{\gamma_{\mathrm{th}}}{f}$ in I_1, we get $u_{\mathrm{yu}} = \frac{\gamma_{\mathrm{th}}}{gf}\left(f + \frac{\gamma_{\mathrm{th}}+1}{\bar{y}}\right)$. Then, solving the integration w.r.t. to the variables u and v, and after some lengthy but straightforward steps, we obtain

$$I_1 = \mathrm{e}^{-\frac{3\gamma_{\mathrm{th}}}{g}} \int_{\frac{\epsilon}{f}}^{\infty} \Big\{\frac{1}{6} + \frac{1}{2}\left[\mathrm{e}^{-\frac{2\gamma_{\mathrm{th}}\tilde{\gamma}_{\mathrm{th}}}{gf\bar{y}}} - \mathrm{e}^{-\frac{\gamma_{\mathrm{th}}\tilde{\gamma}_{\mathrm{th}}}{gf\bar{y}}}\right] - \frac{1}{6}\mathrm{e}^{-\frac{3\gamma_{\mathrm{th}}\tilde{\gamma}_{\mathrm{th}}}{gf\bar{y}}}\Big\} f_{\alpha_{1,\mathrm{d}}}\left(\bar{y} + \frac{\gamma_{\mathrm{th}}}{f}\right) d\bar{y} \quad (29)$$

where $\tilde{\gamma}_{\mathrm{th}} = \gamma_{\mathrm{th}} + 1$. Solving integration over $\bar{y}$ in (29), we obtain I_1. It follows that $\tilde{t}_1 = I_1 + I_2 = 2I_1$. Thus, $\tilde{t}_1$ can be expressed as

$$\tilde{t}_1 = \mathrm{e}^{-\gamma_{\mathrm{th}}\left(\frac{3}{g}+\frac{1}{f}\right)} \left[\frac{1}{3}\mathrm{e}^{-\frac{\epsilon}{f}} + s_2 - s_1 - \frac{1}{3}s_3\right] \quad (30)$$

where

$$s_m = \int_{\frac{\epsilon}{f}}^{\infty} \mathrm{e}^{-\bar{y}} \mathrm{e}^{-\frac{m\gamma_{\mathrm{th}}\tilde{\gamma}_{\mathrm{th}}}{gf\bar{y}}} d\bar{y}, \ m = [1,2,3]. \quad (31)$$

The integral in (31) can be expressed in terms of Whittaker hypergeometric functions [10]. Using series expansion for an exponential function, we can re-express s_m as

$$s_m = \mathrm{e}^{-\frac{\epsilon}{f}} + \sum_{k=1}^{\infty} (-m\gamma_{\mathrm{th}}\tilde{\gamma}_{\mathrm{th}})^k \frac{(gf)^{-k}}{k!} \times \int_{\frac{\epsilon}{f}}^{\infty} \mathrm{e}^{-\bar{y}} \bar{y}^{-k} d\bar{y}. \quad (32)$$

Applying eq. (3.381.6) of [10], s_m can be expressed as in (13).

Appendix B

Substituting the expressions for s_1 and s_2, and after some simple steps, we can express $0.5\mathrm{e}^{-\frac{\epsilon}{f}} + 0.5s_2 - s_1$ as

$$0.5\mathrm{e}^{-\frac{\epsilon}{f}} + 0.5s_2 - s_1 = \sum_{k=2}^{\infty} \frac{2^{k-1}-1}{k!} (-\gamma_{\mathrm{th}}(\gamma_{\mathrm{th}}+1))^k (\epsilon f)^{-\frac{k}{2}} \times \mathrm{e}^{-\frac{\epsilon}{2f}} W_{-\frac{k}{2},\frac{1-k}{2}}\left(\frac{\epsilon}{f}\right) g^{-k}. \quad (33)$$

Similarly, the term $r \triangleq \frac{1}{3}\mathrm{e}^{-\frac{\epsilon}{f}} + s_2 - s_1 - \frac{1}{3}s_3$ can be given by

$$r = \sum_{k=3}^{\infty} \frac{2^k - 1 - 3^{k-1}}{k!} (-\gamma_{\mathrm{th}}(\gamma_{\mathrm{th}}+1))^k \epsilon^{-\frac{k}{2}} \times \mathrm{e}^{-\frac{\epsilon}{2f}} W_{-\frac{k}{2},\frac{1-k}{2}}\left(\frac{\epsilon}{f}\right) f^{-\frac{k}{2}} g^{-k}. \quad (34)$$

On the other hand, $\mathrm{e}^{-\frac{\epsilon}{f}} - s_1$ takes the form

$$\mathrm{e}^{-\frac{\epsilon}{f}} - s_1 = -\sum_{k=1}^{\infty} \frac{1}{k!} (-\gamma_{\mathrm{th}}(\gamma_{\mathrm{th}}+1))^k \epsilon^{-\frac{k}{2}} \mathrm{e}^{-\frac{\epsilon}{2f}} \times W_{-\frac{k}{2},\frac{1-k}{2}}\left(\frac{\epsilon}{f}\right) g^{-k} f^{-\frac{k}{2}}. \quad (35)$$

Substituting (33)-(35) into (23) and then considering only the terms that dominate the outage probability at high SNR (i.e., $e, f \to \infty$), we obtain (24).

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